Thermoemission (dust-electron) plasmas: Theory of neutralizing charges

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Thermoemission plasma—i.e., a system consisting of dust grains and electrons—is studied. In the proposed model, it is assumed that the major part of the electronic gas is uniformly distributed in space and the spatial inhomogeneities of electronic density exist only near the dust grains. The experimental data, well described by the proposed theory, are given.

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I. INTRODUCTION

The combustion of metal powder in an oxygen medium is a perspective method of obtaining of high-purity submicron metal oxides. A low-temperature plasma with condensed dispersed phase [1,2] is formed in the region of condensation of the products of combustion of metal powder cloud burns. Such plasma consists of gas at atmospheric pressure and solid or liquid dust grains, resulting from volume condensation or the particles being of not-burnt fuel. The absolute temperature of such plasma is usually about 1500–3500 K (0.1–0.3 eV), and the system is considered isothermal.

When the plasma contains easily ionizable additional agents of alkali metals with higher number density, the atoms of these alkali metals act as the basic suppliers of free electrons and singly charged positive ions. In such a plasma, the condensed dust grains can be charged both positively and negatively, depending on the relation between the chemical potential of the plasma electrons and the electronic work function from the condensed grain. The interaction between the condensed and gas phases is of complex nature and can lead to the formation of ordered structures [3–5].

When the atoms of alkali metals are the natural impurity, the number of ions in the plasma is negligibly small compared to the number of electrons, formed as a result of thermionic emission from the surface of the condensed dust grains. Such a plasma is called thermoemission plasma. The general feature of thermoemission plasma is the presence of only positively charged dust grains and electrons, emitted by these grains. Therefore, another name for such a system is dust-electron plasma [6,7].

The present paper is dedicated to the development of a theoretical model of thermoemission plasmas, complying with the experimental data, obtained while investigating the plasma formed during the combustion of metal powders.

II. STATE OF THE PROBLEM

The potential spatial distribution around a single dust grain can be described by the Poisson equation

$$\nabla^2\varphi = -4\pi\rho, \tag{1}$$

where φ is the electrostatic potential; ρ is the density of charge, thus $\rho = -eN$ in the space between dust grains in the thermoemission plasma; N is the electron number density.

It is necessary to set the boundary conditions for Eq. (1), which is not an easy problem, like the one Einstein faced in cosmology [8]. When electronic gas is in equilibrium with the condensed grains, the Boltzmann distribution law $N \sim \exp(e\varphi/T)$ for electrons is valid. To the finite difference of potentials there corresponds the finite ratio of electron number densities; therefore, to the zero value of electron number density on the perpetuity there is a corresponding zero density at the grain surface. Otherwise, to the condition $N(\infty) \to 0$ there is a corresponding requirement $\varphi(\infty) \to -\infty$.

This difficulty cannot be overcome within the limits of usual Poisson-Boltzmann theory. Therefore various approaches are used to select the best suitable boundary conditions. For example, the charge density can be represented as a set of electrons subject to the Boltzmann factor and the uniformly distributed positively charged dust grains [9,10]. In Refs. [1,11,12], on the contrary, the Wigner-Seitz model is used, where the system of dust grains is represented as a system of cells with radius

$$R_W = \left(\frac{4}{3}\pi n_d\right)^{-1/3}, \tag{2}$$

where n_d is the number density of the dust grains.

The potential is minimum at the cell boundary and $\varphi'_W = 0$. It is suggested to solve the "flat" task at the grain surface, then make a renormalization of the potential and transfer it to the spherical symmetry. This model excludes the existence of monotonous spatial distribution of the potential, though such a possibility is demonstrated by the numerical modeling.

The attempt to make a model of thermoemission plasma, using charged planes as an example, has been made in Refs. [13,14]. It is supposed that there is such point r_*, where potential $\varphi(r_*) = 0$, and the electron number density in this point complies with condition $(8\pi e^2/T)^3 N_* = 1$. Three kinds of solutions have been considered: the case of semi-infinite plasmas, the case when the minimum potential exists in the center between the planes (it is similar to the model of the Wigner-Seitz cells [1]), and the monotonous potential distribution between the planes. The disadvantage of such a model is the groundlessness of the existence of a nonzero electron density at the point of zero potential.

Thus, one can see that the description of the thermoemission (or dust-electron) plasma is a nontrivial problem. Einstein proposed to modernize the Poisson equation, having postulated that there is some value of the potential φ_0 which corresponds to the uniform distribution $\rho = \rho_0$ in space. In the

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present paper, the model of the thermoemission plasma, based on this suggestion, has been considered.

III. THEORETICAL MODEL

Let us consider the system of condensed dust grains with the same temperature $T \sim 0.3$ eV. The dust grains at such a temperature emit electrons and, in the state of thermodynamic equilibrium, the grain charge and the electron number density at the grain surface are defined by the equality of the fluxes of thermionic emission and uptake of electrons by grain. The surface electron number density is constant and described by the Richardson number density [1]

$$N_s = \nu_e \exp \frac{-W}{T}, \tag{3}$$

where $\nu_e = 2(m_e T/2\pi\hbar^2)^{3/2}$ is the effective density of the electron states, m_e is the electronic mass, $\hbar$ is the Planck constant, and W is the electronic work function from the condensed grain.

The charge of the grain is screened by the electrons; therefore, the potential changes substantially only in the thin layer at the grain surface. Accordingly, the electron number density changes only in this layer. In the rest space between the grains, the electron number density has some constant value N_0, for which there is some corresponding constant value of potential φ_0 [20]. Thus, the potential of the grain surface, with respect to φ_0, is determined by the relation of the surface density Eq. (3) and N_0,

$$\phi_s = \frac{T}{e} \ln \frac{N_s}{N_0}, \tag{4}$$

where $\phi_s = \varphi_s - \varphi_0$.

On the other hand, the potential barrier on the plasma–dust-grain boundary is determined by the difference between the electronic work functions from the dust grain and from the plasma.

The average electron energy is equal to $3T/2$ in the isothermal system. Then, it is possible to determine the formal work function of the electron from the plasma $W_{pl} = 3T/2 - \mu_e$, where $\mu_e = T \ln(\bar{N}/\nu_e)$ is the chemical potential of electrons. Thus, if the electronic work function from the dust grain $W_0 = W_{pl}$, then the potential barrier is absent and in this case the surface density $N_s = N_0$ is the equilibrium value. The uniformly distributed electronic gas would neutralize the charge of the grains with the electronic work function W_0.

Thus, the dust grains with an electronic work function of $W_0 = W_{pl}$ and electronic gas with number density of N_0 create a uniform background of neutralizing charges, which is similar to the model of one-component plasmas [15–19]. If the electronic work function differs from W_0, the electron number density at the grain surface changes, the potential of the surface $\varphi_s \neq \varphi_0$ ($\phi_s \neq 0$) changes too, and there is a field forming a boundary sheath.

Generally, the condensed dust grains are of different sizes and composition. Therefore the charge neutrality of the system is described by the equation

$$\sum_j Z_j n_j = \bar{N}, \tag{5}$$

where Z_j is the charge number of a grain of kind j, with number density n_j; $\bar{N}$ is the average electron number density.

The dust grains with electronic work function $W_j < W_0$ obtain additional positive charge, which provides for the electron surface density $N_{sj} > N_0$, so that the thermionic flux is equal to the flux of electron absorption. For dust grains with electronic work function $W_k > W_0$ the background electron density N_0 is higher than is needed for equilibrium between the electronic gas and the dust grain. Therefore, the grain potential should be less than φ_0, which provides for a field that decreases the absorption of electrons so that the inequality $N_{sk} < N_0$ takes place.

Therefore, the charge density in Eq. (1) can be defined as the deviation of the electron number density from N_0,

$$\rho = -eN_0\left(\exp \frac{e\phi}{T} - 1\right), \tag{6}$$

where $\phi = \varphi - \varphi_0$.

In this case, the Poisson equation for the electronic gas between the dust grains in the dimensionless variables $\Phi = e\phi/T$ and $x = r/\lambda_0$ can be defined in the following form:

$$\nabla^2 \Phi = \exp(\Phi) - 1, \tag{7}$$

where $\lambda_0 = \sqrt{T/4\pi e^2 N_0}$ is the screening length.

The background number density N_0 is always less than the average electron number density $\bar{N}$, as the solution Eq. (7), having a minimum value, cannot cut the direct line φ_0. From Eq. (3) it follows that

$$N_0 = \nu_e \exp \frac{-W_{pl}}{T} = \bar{N} \exp \frac{-3}{2} \cong 0.2\bar{N}. \tag{8}$$

The statement of the problem in the form of Eq. (7) considerably simplifies the choice of boundary conditions. In particular, for a single grain, it is possible to apply boundary conditions in the form $\Phi(\infty) = \Phi'(\infty) = 0$. In this case, it is possible to derive the potential distribution around each dust grain with respect to the bulk potential φ_0.

There is no need to solve Eq. (7) completely to describe the interaction between the condensed dust grains in the thermoemission plasma. It is suffice to determine the charges of the grains and the fields they create, as the force and the direction of interaction are determined by the field.

IV. CHARGES OF CONDENSED DUST GRAINS

The charge of a dust grain is defined by the number of electrons in the Wigner-Seitz cell—i.e., in the sphere with radius R_W [Eq. (2)], which surrounds the grain with radius a_j,

$$Z_j = 4\pi \int_{a_j}^{R_W} r^2 N(r) dr. \tag{9}$$

The electron number density in Eq. (9) consists of the constant component N_0 and the deviation

$$n(r) = \frac{1}{4\pi e r^2} \frac{d}{dr}\left(r^2 \frac{d\phi}{dr}\right). \tag{10}$$

Then, from Eqs. (9) and (10) we obtain

$$Z_j = \frac{4}{3}\pi(R_W^3 - a_j^3)N_0 + \left.\frac{1}{e}\left(r^2\frac{d\phi}{dr}\right)\right|_{a_j}^{R_W}. \tag{11}$$

On the boundary of the Wigner-Seitz cell $\phi'=0$ (or ~ 0); therefore, from Eq. (11) it follows that

$$Z_j = (V_W - V_j)N_0 + (1/e)a_j^2 E_{sj}, \tag{12}$$

where $V_W = 1/n_d$ is the cell volume, $V_j = (4/3)\pi a_j^3$ is the volume of grain j, and E_{sj} is the field near the surface of the grain j. Here, we assume that the difference in sizes of different kinds of grains is small; therefore, the common radius of the Wigner-Seitz cells will not strongly affect the accuracy of calculation.

For the small charges of condensed dust grains it is possible to linearize Eq. (7),

$$\Phi'' + \frac{2}{x}\Phi' = \Phi. \tag{13}$$

The proposed model allows one to use the boundary condition $\Phi(\infty)\to 0$. Therefore, for the single dust grain, the solution of Eq. (13) is the well-known Debye-Hückel potential

$$\phi = \phi_{sj}\frac{a_j}{r}\exp\frac{a_j - r}{\lambda_0}; \tag{14}$$

thus, the field at the grain surface is

$$E_{sj} = \phi_{sj}\left(\frac{1}{a_j} + \frac{1}{\lambda_0}\right) \tag{15}$$

and the grain charge number is

$$Z_j = (V_W - V_j)N_0 + \frac{\phi_{sj}a_j(a_j+\lambda_0)}{e\lambda_0}.$$

For the dust grain with radius $a \gg \lambda_0$, Eq. (7) is reduced to a "flat" form

$$\Phi'' = \exp(\Phi) - 1, \tag{16}$$

whence for $\Phi(\infty)\to 0$, $\Phi'(\infty)\to 0$ we obtain

$$\Phi' = \pm\sqrt{2}\sqrt{\exp(\Phi) - \Phi - 1}. \tag{17}$$

Accordingly, the field at the grain surface

$$E_{sj} = \frac{\sqrt{2}T}{\mathrm{sgn}(\phi_{sj})e\lambda_0}\sqrt{\exp\frac{e\phi_{sj}}{T} - \frac{e\phi_{sj}}{T} - 1}. \tag{18}$$

The numerical simulation demonstrates [5] that a good result, applicable to any radius and charge of the grain, can be obtained by formal integration of Eqs. (15) and (18):

$$E_{sj} = \frac{\sqrt{2}(a_j+\lambda_0)T}{\mathrm{sgn}(\phi_{sj})ea_j\lambda_0}\sqrt{\exp\frac{e\phi_{sj}}{T} - \frac{e\phi_{sj}}{T} - 1}. \tag{19}$$

This expression, at small potentials, is transformed into Eq. (15) and at greater grain radius into Eq. (18). The use of Eq. (19) allows one to determine the grain charge number, taking into account Eq. (4), as

$$Z_j = (V_W - V_j)N_0 + \frac{\sqrt{2}a_j(a_j+\lambda_0)T}{\mathrm{sgn}(\phi_{sj})e^2\lambda_0}\sqrt{\frac{N_s}{N_0} - \ln\frac{N_s}{N_0} - 1}. \tag{20}$$

For the thermoemission plasma the relation $a \ll \lambda_0 \sim R_W$ is typical, accordingly $V_W \gg V_j$. In this case Eq. (20) can be reduced to

$$Z_j \cong \frac{N_0}{n_d} + \frac{\sqrt{2}a_j T}{\mathrm{sgn}(\phi_{sj})e^2}\sqrt{\frac{N_s}{N_0} - \ln\frac{N_s}{N_0} - 1}. \tag{21}$$

For the monodisperse distribution of grains, from Eqs. (5), (8), and (21) we obtain

$$\frac{N_0}{n_d} = 0.4\frac{a_j T}{e^2}\sqrt{\frac{N_s}{N_0} - \ln\frac{N_s}{N_0} - 1}. \tag{22}$$

This equation allows one to determine the background density N_0, after which it is possible to determine the grain charge [Eq. (20)] and the average electron number density [Eq. (8)].

V. COMPARISON TO EXPERIMENTAL DATA

The experimental data [21,22] were obtained in the low-temperature thermal plasma of atmospheric pressure, containing dust grains of cerium dioxide (W=2.75 eV) with the following parameters: absolute temperature 1700 K (T=0.15 eV), grain radius a=0.4 μm, grain number density $n_d = 5\times 10^7$ cm^{-3}, electron number density $\bar{N} \sim 5\times 10^{10}$ cm^{-3}, and average charge number $Z \sim 1000$.

The calculations, mentioned in Ref. [13] using the model of the charged planes, produce the average charge number Z=13, which the author has explained by the uncertainty of the electronic work function and infringement of the thermodynamic equilibrium.

The solution of Eq. (22) gives the background number density $N_0 = 1.15\times 10^{10}$ cm^{-3}; accordingly, $\bar{N} = 5.23\times 10^{10}$ cm^{-3}; the average charge number Z=1046; and the relative surface potential ϕ_s=0.78 V ($5.3T/e$). The work function, corresponding to the neutralization of the electronic gas, is W_0=3.53 eV. Hence, if in such a system there are dust grains with electronic work function over 3.53 eV, their relative potential will be negative. Apparently, the proposed model of neutralizing charges describes well the experimental data in line with the thermodynamic equilibrium.

Let us use the experimental data, obtained in the plasma, which exist in the two-phase flame while burning the metal dust clouds [23]. In this case, a thermoemission plasma is formed in the area of condensation of the metal oxides. The thermoemission plasma of the grains of aluminium oxide (W=4.7 eV) has the following parameters: absolute temperature 3150±70 K (T=0.27 eV), grain radius a=0.05 μm, grain number density $n_d = 10^{10}$ cm^{-3}, and electron number density $\bar{N} \sim 1.5\times 10^{12}$ cm^{-3}.

The solution of Eq. (22) gives the background number density $N_0 = 3.2\times 10^{11}$ cm^{-3}, the grain charge number Z=147, and the relative surface potential ϕ_s=1.1 V ($4.4T/e$). The calculated average electron number density is

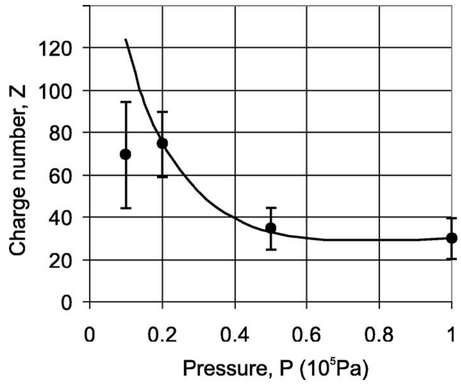

FIG. 1. The calculated (solid line) and measured (dots) (Ref. [24]) charges of the magnesium oxide grains.

$\bar{N}=1.47\times10^{12}$ cm^{-3}, which corresponds to the measuring data.

The following experimental data [24] have been obtained while studying the combustion of magnesium [25] at various pressures: $P=(0.1,0.2,0.5,1)\times10^5$ Pa. In the area of condensation, dust grains of magnesium oxide with average radius $a\sim0.05$ μm are formed. The temperature of the thermoemission plasma was measured using the spectral method, $T/k_B=(2540,2570,2600,2630)$ K. The number densities of the oxide grains are $n_d=(0.02,0.15,2.5,5)\times10^{11}$ cm^{-3}.

The charge of the grains was measured by their deposition on the electrode. The mass of the deposited grains was compared to their average volume. In Fig. 1, the results of measuring of the grain charges and the result of calculation, in which the electronic work function $W=3.9$ eV has been used, are presented.

The electron number density was measured using the probe method at the pressure $P=0.1\times10^5$ Pa: $N_{\text{expt}}=(5\pm3)\times10^{11}$ cm^{-3}. The calculated value at this pressure is $\bar{N}=2.6\times10^{11}$ cm^{-3}.

VI. CONCLUSIONS

The model of neutralizing charges corresponds to the experimental data and can be used to describe the processes of dust grain interactions in the thermoemission (dust-electron) plasma. The mismatch was observed only in one case, which, as the author of the experiment of Ref. [24] explains, results from a great measurement error under conditions of low pressure.

The advantage of the model of neutralizing charges is the use of the bulk potential φ_0, which allows one to describe the polydisperse thermoemission plasma. In this case, grains of each kind are characterized by their own potential distribution with respect to φ_0 [3]. The superposition of the potential distributions around different grains gives a view of the total potential distribution pattern in the system. This distribution can make the field between the grains provide for their attraction or repulsion. If the surface potentials of the two neighboring grains have different signs with respect to the bulk potential, the grains attract. If the relative surface potentials are of identical signs, the grains repulse.

It should be noted that in the considered experimental results, the relative potential of the grains $\phi_s \gg T/e$. It means that the linearization of the Poisson-Boltzmann equation cannot be used to describe real dust-electron systems.

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