Electrostatic interaction of charged planes in the thermal collision plasma: Detailed investigation and comparison with experiment

V. I. Vishnyakov and G. S. Dragan
I. I. Mechnikov Odessa National University, Odessa 65026, Ukraine
(Received 16 June 2004; revised manuscript received 29 September 2004; published 26 January 2005)

The spatial distribution of electrostatic potential between the metal planes in a thermal collision plasma at atmospheric pressure has been investigated. It has been shown that the potential requirement must be calculated with respect to the bulk plasma potential, which depends on the ionization equilibrium in the plasma. It has also been shown that the electrostatic perturbation in the plasma is detected only at distances of less than four screening lengths. Long-range perturbation is described by the bulk plasma potential. The electrostatic pressure on the plane as a function of boundary conditions has been found. The experimental results prove the existence of interaction between the planes, located in the low-temperature plasma at a distance that considerably exceeds the screening length, caused by changing the bulk plasma potential. The application of the results to a complex dusty plasma has shown the diminution of-the dust component dissipation in strongly coupled plasmas.

DOI: 10.1103/PhysRevE.71.016411 PACS number(s): 52.27.Gr

I. INTRODUCTION

The regular spatial distribution of dust grains was demonstrated by experimental research on gas-discharge dusty plasmas [1–3]. The tendency to form ordered structures was also observed in a plasma of solid fuel combustion products [4,5]. High interest in the theoretical exposition of interaction between dust grains has been noted [6–9]. But using the Poisson-Boltzmann theory with reference to the Wigner-Seitz model [10] most authors make serious mistakes already at the stage of assigning of the boundary conditions. A detailed examination of the problem shows that in restricted areas the potential and the electric field cannot be equal to zero at the same time. Moreover, it is impossible to describe the interaction between the grains using such an approach [8]. In the paper [9] a flat model of interaction of charged dust grains has been considered within the Poisson-Boltzmann theory. The search for a solution is based on the assumption that the number densities of electrons and ions in the plasma can be unequal. Such a statement of the problem has allowed for some different solutions to be obtained, though the problem remains unresolved.

The present paper is devoted to a detailed study of grain interaction in an equilibrium plasma using the example of interaction between charged parallel metal planes and the plasma bordered by them, at various values of the electronic work function of the planes and the atom ionization potential of the plasma.

II. STATEMENT OF THE PROBLEM

A. The Poisson-Boltzmann equation

We shall consider an equilibrium plasma, which consists of a buffer gas at atmospheric pressure and easily ionizable addition agents of alkali-metal atoms with number density $N_A = 10^{10}$–10^{17} cm^{-3}. The plasma is in contact with metal parallel planes, which border it. At the isothermal temperature $T = 0.1$–0.3 eV$(T/k_B = 1200$–3500 K$)$ the formation of free electrons in the plasma is ensured by the collision ionizations of the added alkali-metal atoms. If the plasma did not interact with the planes the requirement of neutrality would be satisfied and the total of the ions would be equal to the quantity of electrons. However, the interfacial interaction [11] originally charges the planes and consequently the total of electrons in the plasma will not be equal to the quantity of ions.

The spatial distribution of potential $\varphi(r)$ in the plasma is found using the Poisson-Boltzmann equation,

$$\nabla^2\varphi = 4\pi e[n_{e0}\exp(e\varphi/T) - n_{i0}\exp(-e\varphi/T)], \tag{1}$$

where n_{e0} and n_{i0} are the electron and ion number densities at the point of zero potential.

In Ref. [9] it was generally assumed that $n_{e0} \neq n_{i0}$ and consequently Eq. (1) was reduced to the following form:

$$\nabla^2\varphi = 4\pi e n_{e0}[\exp(e\varphi/T) - \alpha\exp(-e\varphi/T)],$$

where $\alpha = n_{i0}/n_{e0}$.

However, it is possible to act in a different way. It is evident that Eq. (1) has a trivial solution in the case when the potential is equal to some value $\varphi = \varphi_0$, at which $\nabla^2\varphi_0 = 0$,

$$\varphi_0 = (T/2e)\ln(n_{i0}/n_{e0}). \tag{2}$$

Either of the two replacements $\varphi(r) = \varphi_0 \pm \phi(r)$ reduces Eq. (1) to the following form:

$$\nabla^2\phi = 8\pi e\sqrt{n_{e0}n_{i0}}\sinh(e\phi/T). \tag{3}$$

It means that all the solutions of Eq. (1) are symmetrical concerning Eq. (2), and any solution different from the trivial solution cannot touch this value in terms of the theorem of existence and uniqueness. This means that on a restricted area there is no point where $\phi' = \phi = 0$.

Equation (3) is given in the dimensionless form by means of the change of the variables

$$\Phi = e\phi/T, \quad x = r/r_D,$$

where $r_D^2 = T/8\pi e^2\sqrt{n_{e0}n_{i0}}$ is the square of the screening length.

Then for a flat case from Eq. (3) it acquires the form of a nonlinear second-order differential equation,

$$d^2\Phi/dx^2 = \sinh(\Phi). \tag{4}$$

The constant value $\varphi_0 \equiv \varphi_{pl}$ is called the bulk potential of the plasma and is necessary for establishing a correlation between different solutions of the Poisson-Boltzmann equation carried out within the Wigner-Seitz model for separate dust grains [12,13].

B. Boundary conditions

In order to define the boundary conditions we shall consider the interphase exchange of electrons on the plane surfaces. The thermionic emission is described by the Richardson-Dushman equation

$$j_e^T = -(4\pi e m_e T^2/(2\pi\hbar)^3)\exp(-W/T),$$

where j_e^T is the density of electronic current into the plasma and W is the electronic work function.

We study the equilibrium contact; therefore in agreement with the principle of detailed equilibrium, the electronic back flow stipulated by the random motion of plasma electrons corresponds to the thermionic current [11],

$$j_e^{abs} = (1/4)en_{es}\overline{C_e},$$

where $\overline{C_e} = \sqrt{8T/\pi m_e}$ is the thermal velocity of electrons and n_{es} is the number density of electrons at the plane surface.

The sum of these currents allows calculating the equilibrium value of the surface electron number density

$$n_{es} = \nu_e \exp(-W/T),$$

where $\nu_e = 2(m_e T/2\pi\hbar^2)^{3/2}$ is the effective density of states of electrons.

The equilibrium electron number density near the metal surface n_{es} does not depend on ionization processes in plasma and is defined only by saturation pressure of electrons. In the volume of plasma the ionization equilibrium is defined by the Saha equation [14],

$$n_q^2 = n_e n_i = n_a (g_i/g_a)\nu_e \exp(-I^*/T),$$

where n_q is the quasiunperturbed density, g_i and g_a are the statistical weights of ions and atoms, and I^* is the effective ionization potential of the added atoms, which may differ from the real ionization potential of an isolated atom.

The relation between n_{es} and n_q determines the charge of the plane, because, according to the Boltzmann factor, $n_{es} = n_q \exp(e\phi_s/T)$, where ϕ_s is the potential of the surface. If the equilibrium at the boundary between the metal plane and the plasma demands such a value of surface electron density as $n_{es} < n_q$, then the plane is charged negatively, thus diminishing the probability of collision with the plasma electrons. Otherwise the plane is charged positively. Finally all is reduced to the relation between the work function and the ionization potential,

$$e\phi_s = I^*/2 - W + (T/2)\ln(g_a\nu_e/g_i n_a). \tag{5}$$

Therefore, only the values of the potential on the planes ϕ_1 and ϕ_2, to which correspond dimensionless values Φ_1 and Φ_2, can be used as boundary conditions. These values cannot be arbitrary, but are defined according to Eq. (5) by the interaction between the planes and the plasma. We note that the surface value of the potential ϕ_s is calculated with respect to φ_{pl}. The total value of the potential on the plane surface is $\varphi_s = \varphi_{pl} + \phi_s$.

III. THE DISTRIBUTION OF POTENTIAL

A. Preliminary study

We shall pay attention to the fact that, in view of the odd function $\sinh(x)$, the solution of Eq. (3) has a local minimum, located in the half plane $\varphi > \varphi_{pl}$, and the solution has a local maximum, located in the half plane $\varphi < \varphi_{pl}$. Hence, if the planes are given values of the total potential φ_1 and φ_2, so that $\mathrm{sgn}(\varphi_1 - \varphi_{pl}) = \mathrm{sgn}(\varphi_2 - \varphi_{pl})$ is true, then the spatial distribution $\varphi(r)$, having in this case an extreme value, does not cross the line φ_{pl}, and for the electric field on the planes $\mathrm{sgn}(\mathbf{E}_1) = -\mathrm{sgn}(\mathbf{E}_2)$ is true. If the condition $\mathrm{sgn}(\varphi_1 - \varphi_{pl}) = -\mathrm{sgn}(\varphi_2 - \varphi_{pl})$ is valid, then the spatial distribution $\varphi(r)$ is a monotonically decreasing or monotonically increasing function, and for the electric field, in this case, $\mathrm{sgn}(\mathbf{E}_1) = \mathrm{sgn}(\mathbf{E}_2)$ is valid.

Thus, Eq. (3) has two sets of solutions, and the character of the solutions depends on the relation between the boundary conditions φ_1, φ_2 and the bulk plasma potential φ_{pl}, which is determined by the ionization state of the plasma, Eq. (2).

This implies, that it is impossible to neglect the constant value φ_{pl} as is usually done, because the constant value of the potential does not influence the interaction of dust grains and is determined by the electric field only. However, if we decide to measure the potential of the planes, the zero potential will already be given, being the potential of the ground. Suppose the measured values of the potential are $\varphi_1 > \varphi_2 > 0$. Beforehand we may not say that $\mathrm{sgn}(\mathbf{E}_1) = -\mathrm{sgn}(\mathbf{E}_2)$ and the planes repel, as is clear from the analysis above. The ionization state of the plasma may be such that the value φ_{pl} ensures $\varphi_1 > \varphi_{pl} > \varphi_2$. Then $\mathrm{sgn}(\mathbf{E}_1) = \mathrm{sgn}(\mathbf{E}_2)$ and the planes are attracting.

B. The solutions

Let us lower the order of Eq. (4):

$$\Phi' = \pm 2\sqrt{\sinh^2(\Phi/2) + \delta}. \tag{6}$$

The value of the constant $\delta = 0$ produces Eq. (6) in the following form:

$$\Phi' = -2\sinh(\Phi/2), \tag{7}$$

which corresponds to a semi-infinite volume of the plasma, provided that $\Phi(\infty) = 0$.

The solution Eq. (7) is the function [6]

$$\Phi = 2\ln\left[\frac{1 + \tanh(\Phi_1/4)\exp(x_1 - x)}{1 - \tanh(\Phi_1/4)\exp(x_1 - x)}\right]. \tag{8}$$

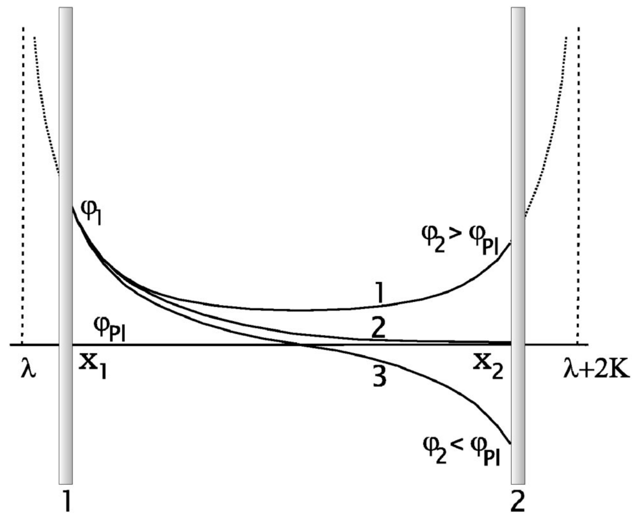

FIG. 1. Geometry of the task.

The solutions expressed in elliptic functions, whose qualitative forms are given in Fig. 1, correspond to the values of the constant $\delta \neq 0$. In order to find these solutions it is necessary to carry out a series of transformations. First of all, we note that Eq. (6) is an equation with separable variables and may be given as the integral

$$\int_{\Phi}^{\infty} \frac{d\Phi}{\sqrt{\sinh^2(\Phi/2)+\delta}} = \pm 2(\lambda - x). \tag{9}$$

Here λ is a value of the coordinate (on the left, before the plane r_1—see Fig. 1) which approaching the potential asymptotically tends to infinity. Such a choice of the limits of integration is caused by the necessity of reduction of the integral (9) to a canonical form by means of the replacement $t=\sinh(\Phi/2)$,

$$\int_{t}^{\infty} [(t^2+1)(t^2+\delta)]^{-1/2} dt = x - \lambda. \tag{10}$$

The solutions of Eq. (10) are represented according to [15] in Jacobi elliptic functions, namely, for positive values of δ,

$$\sinh\frac{\Phi}{2} = \mathrm{sgn}(\Phi_1)\frac{\mathrm{cn}(x-\lambda, 1-\delta)}{\mathrm{sn}(x-\lambda,1-\delta)},$$

$$\frac{d\Phi}{dx} = -2\,\mathrm{sgn}(\Phi_1)\frac{\mathrm{dn}(x-\lambda,1-\delta)}{\mathrm{sn}(x-\lambda,1-\delta)}, \quad \delta < 1,$$

$$\sinh\frac{\Phi}{2} = \mathrm{sgn}(\Phi_1)\sqrt{\delta}\,\frac{\mathrm{cn}(\sqrt{\delta}(x-\lambda),1-1/\delta)}{\mathrm{sn}(\sqrt{\delta}(x-\lambda),1-1/\delta)},$$

$$\frac{d\Phi}{dx} = -2\,\mathrm{sgn}(\Phi_1)\sqrt{\delta}\,\frac{\mathrm{dn}(\sqrt{\delta}(x-\lambda),1-1/\delta)}{\mathrm{sn}(\sqrt{\delta}(x-\lambda),1-1/\delta)}, \quad \delta > 1. \tag{11}$$

For negative values of δ,

$$\sinh\frac{\Phi}{2} = \mathrm{sgn}(\Phi_1)\sqrt{1-\delta}\,\frac{\mathrm{dn}(\sqrt{1-\delta}(x-\lambda),1/(1-\delta))}{\mathrm{sn}(\sqrt{1-\delta}(x-\lambda),1/(1-\delta))},$$

$$\frac{d\Phi}{dx} = -2\mathrm{sgn}(\Phi_1)\sqrt{1-\delta}\,\frac{\mathrm{cn}(\sqrt{1-\delta}(x-\lambda),1/(1-\delta))}{\mathrm{sn}(\sqrt{1-\delta}(x-\lambda),1/(1-\delta))}. \tag{12}$$

C. The limiting thickness of the plasma layer

Equations (11) and (12) feature periodic functions with the period $4K$, where K is the complete elliptic integral of the first kind,

$$K(m) = \int_0^1 [(1-t^2)(1-mt^2]^{-1/2} dt, \tag{13}$$

where $m=1-\delta$ for $1>\delta>0$ and $m=1/(1-\delta)$ for $\delta<0$.

This means that the functions (11) and (12) reach infinite values at the distance of $2K$ from λ. Therefore, the relations must be evaluated for the thickness of the plasma layer between the planes $L=r_2-r_1$,

$$L/r_D < 2K, \quad \delta > 0,$$

$$L/r_D < K(2/(1-\delta)), \quad \delta < 0.$$

The thickness of the plasma layer is interesting in many respects, as it is much greater than the screening length, i.e., $L/r_D \gg 1$, which corresponds to $K \gg 1$. As it follows from the tables [15] for the complete elliptic integrals of the first kind, the values of the parameter $m \cong 1$ correspond to the major values of K. For example, the value of the parameter $m = 0.995$ corresponds to the value $K=4$, whence we obtain the value of the constant $\delta = \pm 0.005$. At such small values of the constant δ it can be neglected in Eq. (6). Thus, if the thickness of the layer $L \geqslant 8r_D$, the spatial potential distribution near the plane, with adequate accuracy, is described by the function (8) for a semi-infinite plasma which can be given in the following form:

$$\Phi = 2\ln\left[\frac{1+\mathrm{sgn}(\Phi_1)\exp(\lambda - x)}{1-\mathrm{sgn}(\Phi_1)\exp(\lambda - x)}\right], \tag{14}$$

where the constant $\lambda = x_1 + \ln|\tanh(\Phi_1/4)|$.

In this case it is easy to spot that the potential varies from infinity up to the value $10T/e$ at the distance of $0.013r_D$, up to the value $1T/e$ at the distance of $1.4r_D$, and at the distance of $4r_D$ the potential varies from infinity up to the value $0.07T/e$. This means that if the distance from the plane is more than $4r_D$ the electrical interaction is essentially missing irrespective of the value of the surface potential. Hence, the thickness of the layer $4r_D$ is the limiting distance for propagation of electrostatic perturbation in the plasma.

D. Approximation of solutions

If the layer of plasma is $\geqslant 4r_D$ then the small value of the constant δ allows approximating Eqs. (11) and (12) using the trigonometric functions [16]. The result obtained is as follows:

$$\sinh\left(\frac{\Phi}{2}\right) = \text{sgn}(\Phi_1)\frac{\pi}{2K}\cot\left(\frac{\pi(x-\lambda)}{2K}\right), \quad 1 \geqslant \delta > 0,$$

$$\sinh\left(\frac{\Phi}{2}\right) = \text{sgn}(\Phi_1)\frac{\pi\sqrt{1-\delta}}{2K}\left[\sin\left(\frac{\pi\sqrt{1-\delta}(x-\lambda)}{2K}\right)\right]^{-1},$$

$$0 > \delta \geqslant -1.$$

However, it is more convenient to use a superposition of the solutions for a semi-infinite plasma, Eq. (14), which it is possible to represent as

$$\Phi_L = -2\,\text{sgn}(\Phi_1)\ln\left(\tanh\frac{x-\lambda}{2}\right),$$

$$\Phi_R = -2\,\text{sgn}(\Phi_2)\ln\left(\tanh\frac{\lambda+2K-x}{2}\right).$$

Then the spatial distribution of the potential between planes can be described by the following expression:

$$\Phi = \ln\left[\left(\tanh\frac{x-\lambda}{2}\right)^{k_1}\left(\tanh\frac{\lambda+2K-x}{2}\right)^{k_2}\right], \qquad (15)$$

provided $K \geqslant 4$, $k_1 = -2\,\text{sgn}(\Phi_1)$, and $k_2 = -2\,\text{sgn}(\Phi_2)$.

E. Definition of the constants

The constant λ is defined by the first boundary condition Eq. (14). The constant δ must be defined in two ways: for negative and positive values of the constant. In order to define the negative δ_- we take into account that in this case there is a minimum of the spatial potential distribution Φ_0. From Eq. (6) it follows that

$$\delta_- = -\sinh^2(\Phi_0/2).$$

The minimum Φ_0 is at the distance K from the coordinate λ. Let us define the interval $2K$. On its left this interval is limited by the coordinate λ. Its right coordinate $\lambda + 2K$ is defined by application of Eq. (14) to the second plane, whence it follows that $\lambda + 2K = x_2 - \ln|\tanh(\Phi_2/4)|$, i.e.,

$$2K = x_2 - x_1 - \ln|\tanh(\Phi_1/4)\tanh(\Phi_2/4)|. \qquad (16)$$

The value Φ_0 is determined by Eq. (15). As a result we obtain

$$\delta_- = \frac{-4\cosh^2(K)}{\sinh^4(K)}. \qquad (17)$$

For the positive value of $\delta = \delta_+$ the function $\Phi(x)$ crosses the zero point at the distance K from λ, and it follows from Eqs. (6) and (15) that

$$\delta_+ = (\Phi_0'/2)^2 = 4/\sinh^2(K). \qquad (18)$$

We see that both Eqs. (17) and (18) coincide at major values of K, when $\cosh(K) \cong \sinh(K)$. Therefore we may state that

$$\delta \cong -4\,\text{sgn}(\Phi_1)\,\text{sgn}(\Phi_2)/\sinh^2(K), \qquad (19)$$

because at $|\delta| \ll 1$ the absolute value of the constant δ does not change with only a change of the sign of Φ_1 or Φ_2.

IV. THE BULK PLASMA POTENTIAL

A. Definition of the bulk plasma potential

The obtained solutions of the Poisson-Boltzmann equation feature a spatial distribution of the potential about some value φ_{pl}. As follows from Eq. (2), this value of the potential is determined by the relation of the ion and electron number densities at the zero point of the total potential $\varphi = 0$. Let us take the value of the floating potential of the probe, located in the neutral gas plasma without dust grains, as the zero point (certainly, the measuring potential of the probe with respect to the ground will be distinct from zero because the probe itself is a perturbing factor in the plasma). If we add dust grains to the plasma, the interphase interaction may cause a change of the charged state of the plasma and then the probe will show some other value of the potential. The difference between the previous value and the present value is the bulk plasma potential φ_{pl} itself. If the interphase interaction does not change the charge of the plasma volume, then $n_{i0} = n_{e0}$ and the bulk plasma potential $\varphi_{pl} = 0$.

Thus, the bulk potential of the plasma characterizes the size of operation that is necessary for the plasma to gain some volumetric charge Q_{pl}. The bulk plasma potential and the volumetric charge determine the electrostatic energy of the plasma volume,

$$\mathcal{E} = \frac{1}{2}Q_{pl}\varphi_{pl}.$$

On the other hand, the energy of the electric field produced is determined by the equation

$$\mathcal{E} = \frac{1}{8\pi}\int_V E^2 dV,$$

where E is the electric field.

Hence, the bulk plasma potential in the layer between the planes is equal to

$$\varphi_{pl} = \frac{\int_{r_1}^{r_2} E^2 dr}{4\pi e\int_{r_1}^{r_2}(n_i - n_e)dr} = -\frac{T}{e}\frac{\int_{x_1}^{x_2}(\Phi')^2 dx}{\Phi_2' - \Phi_1'}. \qquad (20)$$

B. Calculation of the bulk plasma potential

We shall consider a layer of plasma with the thickness of $L > 8r_D$. Then for each of the planes it is possible to use the solution Eq. (14) and Eq. (7) for the derivative; then from Eq. (20) we obtain

$$\varphi_{pl} = -2\frac{T}{e}\frac{\coth(\lambda + 2K - x_2) - \coth(x_1 - \lambda)}{\sinh(\Phi_2/2) - \sinh(\Phi_1/2)}. \qquad (21)$$

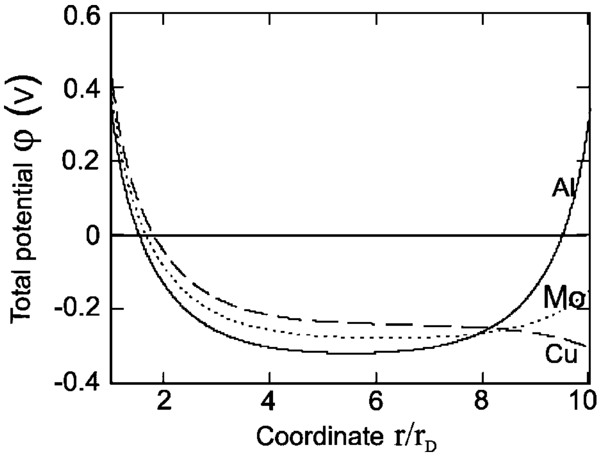

FIG. 2. The spatial distribution of the total potential.

Equation (14) can be converted to the form $\exp(\lambda - x) = \mathrm{sgn}(\Phi_1)\tanh(\Phi/4)$, whence we obtain $\coth(x-\lambda) = \cosh(\Phi/2)$ for the left plane and $\coth(\lambda + 2K - x) = \cosh(\Phi/2)$ for the right plane. Then from Eq. (21) it follows that

$$\varphi_{pl} = -2\frac{T}{e}\frac{\cosh(\Phi_2/2) - \cosh(\Phi_1/2)}{\sinh(\Phi_2/2) - \sinh(\Phi_1/2)} = -2\frac{T}{e}\tanh\left(\frac{\Phi_1 + \Phi_2}{4}\right).$$

The bulk plasma potential in dimensional variables for a layer is,

$$\varphi_{pl} = -2\frac{T}{e}\tanh\left(\frac{e\phi_1 + e\phi_2}{4T}\right), \qquad (22)$$

and taking into account Eq. (5) for the surface potential,

$$\varphi_{pl} = -2\frac{T}{e}\tanh\left[\frac{I^* - W_1 - W_2}{4T} + \frac{1}{4}\ln\left(\frac{g_a \nu_e}{g_i n_a}\right)\right].$$

We note that the potential of the plasma has a sign that is opposite to the sign of the surface potentials. Therefore the total potential is always less, in terms of absolute value, than the Poisson-Boltzmann equation solutions.

Let us consider a specific example, when aluminum planes ($W = 3.74$ eV) are located in potassium plasma ($I = 4.34$ eV) with the number density of added agent $N_A = 10^{15}$ cm^{-3} at the temperature $T = 0.17$ eV($T/k_B = 2000$ K). In this case the effective density of electron states $\nu_e = 4.3 \times 10^{20}$ cm^{-3}. In equilibrium the relative potential of the planes is equal to $\phi_1 = \phi_2 = 0.67$ V and the bulk plasma potential is equal to $\varphi_{pl} = -0.33$ V. Hence, the total potential of the planes is $\varphi_1 = \varphi_2 = 0.34$ V. This value reflects the measurement with respect to the neutral plasma. The calculated spatial potential distribution is given in Fig. 2.

Let us replace the second plane by a molybdenum one ($W = 4.27$ eV). In this case the relative values of the surface potential are also positive: $\phi_1 = 0.67$ V, $\phi_2 = 0.14$ V. The bulk plasma potential is $\varphi_{pl} = -0.28$ V and the measuring instrument will show values of the total potential of a different sign, $\varphi_1 = 0.39$ V and $\varphi_2 = -0.14$ V, though a plot of the spatial potential distribution in the first and second cases corresponds to the value $\delta < 0$ (Fig. 2, Al and Mo), and the planes must be repulsive.

V. INTERACTION OF PLANES

A. Conditions of attraction and repulsion of planes

In [17] the existence of forces between thin metal and dielectric films in the air gas-discharge plasma at pressures of 1.3–13 Pa has been experimentally shown.

The interaction between the planes depends on the form of the Poisson-Boltzmann equation solution. The repulsion of planes occurs at values of the constant $\delta < 0$. For example, in the case of two aluminum planes, as has been observed, and in the case of an aluminum plane and a molybdenum plane, both correspond to negative values of δ, i.e., the planes are repulsing. The change from repulsion to attraction of planes occurs at $\delta = 0$. If the relative potentials of planes are of different signs, the planes will attract. Such a situation occurs if the material of the second plane is copper ($W = 4.47$ eV). In this case the relative potentials have different signs, $\phi_1 = 0.67$ V, $\phi_2 = -0.06$ V, and the spatial distribution of potential between the planes is represented by a monotonically decreasing function (Fig. 2, Cu). The derivative of the potential does not become zero anywhere between the planes and ensures the existence of a unidirectional force attracting the planes.

If we take the first plane in Fig. 1, the electric field on the left surface E_{left} is different from the electric field on the right surface E_{right}, because the plasma is unlimited on its left ($\delta = 0$) and limited on its right ($\delta \neq 0$). This gives rise to the electrostatic pressure P on the plane [9,18]

$$P = \frac{1}{8\pi}(E_{left}^2 - E_{right}^2),$$

$$P = \frac{1}{8\pi}\left(\frac{T}{er_D}\right)^2 [(\Phi')^2_{left} - (\Phi')^2_{right}]$$

and hence, from Eq. (6) we obtain

$$P = -\frac{1}{8\pi}\left(\frac{T}{er_D}\right)^2 \delta, \qquad (23)$$

where δ is determined by Eq. (19), which can be transformed, if $r_2 - r_1 \gg r_D$, to

$$P = \frac{2}{\pi}\left(\frac{T}{er_D}\right)^2 \frac{\tanh\left(\frac{e\varphi_1 - e\varphi_{pl}}{4T}\right)\tanh\left(\frac{e\varphi_2 - e\varphi_{pl}}{4T}\right)}{\exp\left(\frac{r_2 - r_1}{r_D}\right)}. \qquad (24)$$

The dependence of the electrostatic pressure on the relative potential of the second plane and the distance between the planes is shown in Fig. 3. The relative potential of the first plane is 5. Dependence 1 is calculated by means of the computer model using Eq. (23) and dependence 2 is calculated using Eq. (24). We can see that Eq. (24) satisfactorily

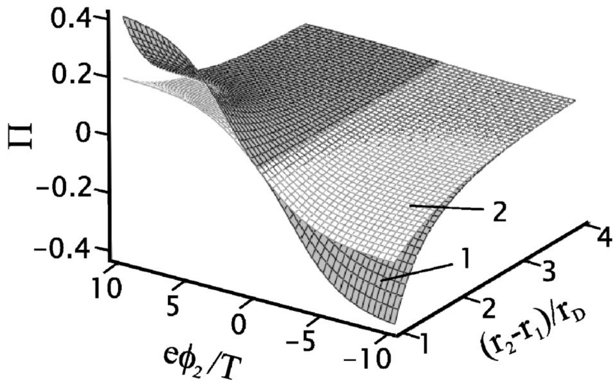

FIG. 3. The dependence of the dimensionless electrostatic pressure $\Pi=(er_D/T)^2P$ on the relative potential of the second plane ϕ_2 and the distance between the planes r_2-r_1. Surface 1 is the result of the computer model; surface 2 is the solution of Eq. (24).

describes the electrostatic pressure if the distance between the planes is more than $3r_D$.

B. Small values of the potential

Let us consider the case of the relative surface potentials $e(\phi_1+\phi_2)\ll 4T$ when Eq. (22) for the bulk plasma potential is linearized: $\varphi_{pl}\cong(\phi_1+\phi_2)/2$. Then the definition of the replacement $\varphi(r)=\varphi_{pl}+\phi(r)$ produces the total surface potentials

$$\varphi_1=(\phi_1-\phi_2)/2, \quad \varphi_2=(\phi_2-\phi_1)/2.$$

This means that $\varphi_1=-\varphi_2$, i.e., the measuring device will show opposite values of the planes' potentials with respect to the neutral plasma at any values of $\phi_1\ll T/e$ and $\phi_2\ll T/e$. Accordingly, the Poisson equation can have any form of solution and the planes can either attract or repel, as determined by the relation of φ_1 and φ_2 to the bulk plasma potential.

VI. THE EXPERIMENT

We have found that the measured value of the potential depends on the values of the potential barriers on the plasma-plane boundaries. Hence, if the measurement of a single electrode potential gives the value φ_s, the introduction of a second electrode should change this value.

A propyl hydride–air flame at the absolute temperature of about 1200 K was used in the experiment. A 40% water solution of potassium carbonate was injected into the air stream. It provided for a potassium number density of $N_A=10^{10}$–10^{11} cm^{-3}. In such conditions the electron and ion equilibrium number densities are $n_q\approx 10^6$ cm^{-3}. Then a flat copper electrode (1×1 cm^2) with a thermoelectric couple is inserted into the flame along the stream. The measured values of the floating potential of the single copper electrode with respect to the ground are given in Fig. 4.

Equation (5) cannot be used for calculation of the floating potential in this case, because the low unperturbed number density n_q does not ensure sufficient electron current. Only

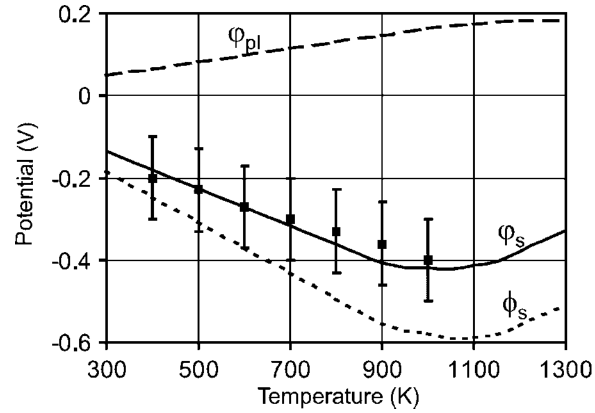

FIG. 4. The calculated dependencies of the surface relative potential ϕ_s, the total potential φ_s, the bulk plasma potential φ_{pl}, and the experimental points on the temperature of the single copper electrode.

the absence of the electric current through the electrode surface should be taken into account, i.e.,

$$j_e^T+j_e^{abs}+j_i^{rec}+j_a^{ion}=0, \qquad (25)$$

where $j_e^T=-(4\pi em_eT^2/(2\pi\hbar)^3)\exp(-W/T)$ is the thermionic emission current density; $j_e^{abs}=(1/4)en_{es}\overline{C_e}$ is the electron absorption current density, $j_i^{rec}=-(1/4)\gamma_s en_{is}\overline{C_i}$ is the current density of the surface recombination of ions, where $\overline{C_i}$ is the thermal velocity of ions, $n_{is}=n_q\exp(-e\phi_s/T)$ is the surface number density of ions, and γ_s is the surface recombination coefficient; $j_a^{ion}=(1/4)\beta_s en_{as}\overline{C_a}$ is the current density of the surface ionization of atoms, where $\overline{C_a}$ is the thermal velocity of atoms $(\overline{C_i}\cong\overline{C_a})$, $n_{as}=N_A-n_{is}$ is the surface number density of atoms, and β_s is the surface ionization coefficient.

The surface ionization coefficient determines the probability of ionization of atoms on the electrode surface [11],

$$\beta_s=\frac{\exp(e\phi_s/T)}{1+(g_a/g_i)\exp[(I^*-W)/T]}.$$

Accordingly, the surface recombination coefficient is

$$\gamma_s=\frac{1}{1+(g_i/g_a)\exp[(W-I^*)/T]}.$$

The solution of Eq. (25) produces the floating potential ϕ_s with respect to the bulk plasma potential. According to Eq. (22) the bulk plasma potential is

$$\varphi_{pl}=-2\frac{T}{e}\tanh\left(\frac{e\phi_s}{4T}\right)$$

and the total potential is $\varphi_s=\phi_s+\varphi_{pl}$.

All these dependencies are shown in Fig. 4. We can see good concurrence of the measurement data and the calculations.

In the second stage of our experiment we leave the copper electrode at the temperature of about 1020 K in the flame and insert a similar aluminum electrode into the flame at the distance of 3 cm from the first one (the screening length here

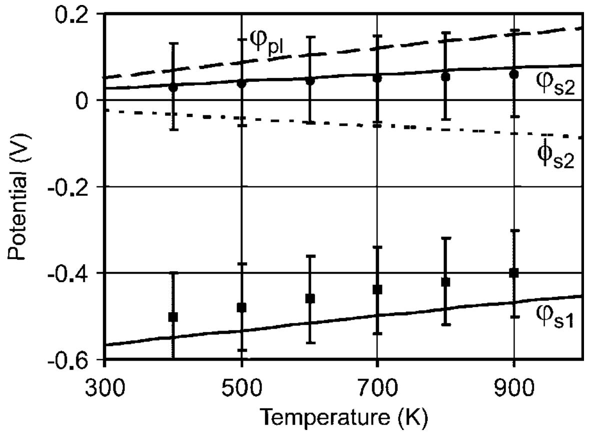

FIG. 5. The calculated dependencies of the surface relative potential ϕ_{s2}, the total potential φ_{s2}, the bulk plasma potential φ_{pl}, and the experimental points on the aluminum electrode temperature; the calculated curve and the experimental points of the total potential φ_{s1} of the copper electrode at the absolute temperature of 1020 K.

is $r_D \approx 0.1$ cm). In this case the relative surface potential of the aluminum electrode ϕ_{s2} is calculated as the solution of Eq. (25), but the bulk plasma potential is defined by both potentials $\phi_{s1}(1020\ \text{K})$ and $\phi_{s2}(T)$. In Fig. 5 we can see that different signs of the relative and total potentials of the aluminum probe are produced. We can also see that the change of the bulk plasma potential causes change of the total potential of the first copper electrode to 0.1 V, i.e., the second isolated electrode inserted into the flame influences the measurement of the first electrode potential.

VII. APPLICATION TO DUSTY PLASMAS

The investigation of electrostatic plane interaction described above can be applied to dusty plasmas when defining the dynamic characteristics of the dust grain interaction. The dynamic properties of the dust component are determined by three dynamic parameters [19]: the nonideality parameter (the Coulomb coupling parameter) $\Gamma = e^2Z^2/R_WT$; the structure parameter $\kappa = R_W/r_D$; the dynamical parameter $\theta = \eta/\omega_d$. Here Z is the charge number of the dust grain, $R_W = (4\pi n_d/3)^{-1/3}$ is the Wigner-Seitz radius, η represents the damping rate associated with the surrounding medium [20], and ω_d is the frequency describing the dust component. The plasma investigated by us corresponds to the case of a complex plasma, therefore the normalized nonideality parameter $\Gamma^* = \Gamma(1+\kappa+\kappa^2/2)^{1/2}\exp(-\kappa)$ should be used [21].

A gas containing the neutral buffer component (air), partially ionized potassium addition agent $N_A = 10^{15}$ cm^{-3}, and grains with radius $a \gg r_D$ at the equilibrium temperature $T = 0.17$ eV have been considered. In this case the Debye screening length is $r_D \cong 1.6$ μm; therefore it is possible to choose the radius of dust grains $a = 10$ μm. At the number density of grains $n_d = 5\times10^7$ cm^{-3} the Wigner-Seitz radius is $R_W \cong 16$ μm and the structure parameter is $\kappa \cong 10$. The charge of the dust grains is defined by the following expression [6]:

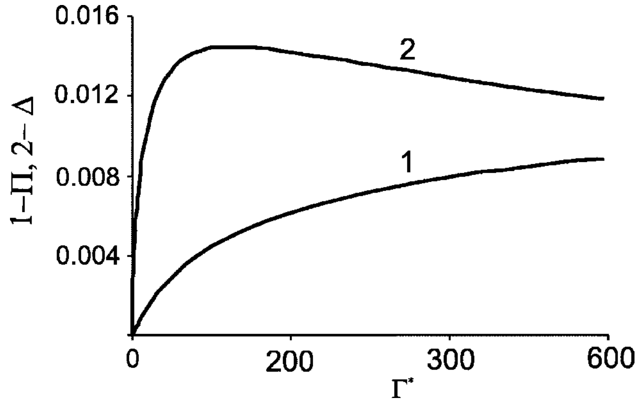

FIG. 6. The dependencies of $\Pi = (er_D/T)^2P$ (curve 1) and $\Delta = 10(er_D^2/T)(\delta P/\delta\phi_s)$ (curve 2) on the normalized nonideality parameter Γ^*.

$$Z = \frac{2a^2T}{e^2r_D}\sinh\left(\frac{e\phi_s}{2T}\right). \tag{26}$$

It is possible to build the dependence of the electrostatic pressure between the two grains using the nonideality parameter, using Eq. (24) and Eq. (26), provided that both grains have identical charges (Fig. 6, curve 1). We can see that the pressure tends to some constant value as the parameter Γ^* increases. Consequently, in a strongly nonideal plasma the interaction between the dust grains is weakly dependent on the fluctuation of the charge (potential) on the grain surfaces. In order to prove it, let us examine the variation of the pressure on the surface potential of one of the grains, using Eq. (24):

$$\frac{\delta P}{\delta\phi_s} = \frac{T}{2\pi e r_D^2}\frac{\tanh(e\phi_s/4T)}{\cosh^2(e\phi_s/4T)}e^{-L}, \tag{27}$$

where $L = 2(R_W - a)/r_D$ is the distance between the surfaces of grains with respect to the screening length.

The result is shown in Fig. 6 (curve 2). Here it is evident that at small values of the nonideality parameter the interaction between the grains increases with the growth of Γ^*. In this case Eq. (27) can be approximated as follows:

$$\frac{\delta P}{\delta\phi_s} \cong \frac{1}{8\pi}\frac{\phi_s}{r_D^2}e^{-L}, \quad \Gamma^* < 1. \tag{28}$$

However, in a strongly nonideal plasma the interaction of grains decreases,

$$\frac{\delta P}{\delta\phi_s} \cong \frac{T}{2\pi e r_D^2}\frac{e^{-L}}{\cosh^2(e\phi_s/4T)}, \quad \Gamma^* \gg 1. \tag{29}$$

This means that in an ideal plasma the oscillation of one dust grain strongly influences the oscillation of other grains. As a result, all the dust components can be shaken by a single fluctuation. In a strongly nonideal plasma the oscillation of one dust grain weakly influences the oscillation of other grains; therefore each dust grain oscillates about an equilibrium state irrespective of the other grains. Thus if the tendency to form structures is characteristic of plasmas [22], extending the oscillations destroys it in systems with small

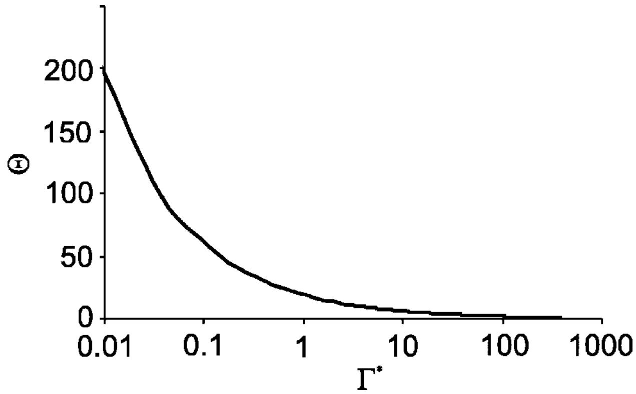

FIG. 7. The dependence of the dynamical parameter θ on the normalized nonideality parameter Γ^*.

Coulomb coupling. This destroying factor disappears in strongly coupled systems, which cause the occurrence of regular structures in the dust [5].

The position of the maximum of curve 2 in Fig. 6 corresponds to the value of the surface potential of the dust grains ϕ_{s0}, which is defined by the derivative of Eq. (27) equaling zero: $\sinh^2(e\phi_{s0}/4T)=1/2$. In our case $\phi_{s0}\cong 0.5$ V, to which correspond the charge number $Z_0\cong 3\times 10^4$ and the nonideality parameter $\Gamma_0^*\cong 100$.

Let us consider the behavior of the dynamical parameters. As the damping rate η we can use the technical formula [21] η (s^{-1})CP_n (Torr)/[a (μm)$\times\rho$ (g/cm^3)], where P_n is the gas pressure (in our case it is the atmosphere), ρ is the grain density (3.5 g/cm^3), and C is a dimensionless parameter, defined by the nature of the neutral gas (for nitrogen $C=400$ is used). For atmospheric pressure we obtain $\eta\cong 8.5\times 10^3$ s^{-1}. For the frequency ω_d we use the equation $\omega_d=\sqrt{4\pi e^2 Z^2 n_d/m_d}$ [20].

As is evident from Fig. 7, the dynamical parameter decreases with the growth of Γ^*. Thus, if the ideal plasma is a dissipative system, then the parameter $\theta\to 0$ in a strongly nonideal plasma, and the dusty plasma can be considered as a nondissipative system. It corresponds to the behavior shown by the dependence in Fig. 6. The reduction of grain interaction corresponds to the reduction of dissipation.

VIII. CONCLUSION

In a thermal collision plasma it is impossible to set arbitrary boundary conditions for the Poisson-Boltzmann equation in order to calculate the dust grain interaction. The boundary conditions are defined by the surface characteristics of the dust grains and the ionization state of the plasma, which may be described using the bulk plasma potential. Thus, the problem is self-consistent not only within the definition of the electric potential and the charge density, but also within the definition of the boundary conditions. The values of the surface potential are defined by the balance of the streams of gas particles on the phase boundary, where the potential is calculated with respect to the bulk plasma potential.

The character of the interaction of the charged dust grains, as shown in the example with the two planes, is defined by the form of the solution of the Poisson-Boltzmann equation. Thus, their attraction or repulsion depends on the sign of the constant δ, which is a function of the surface value of the relative potential.

The electrostatic perturbation in the plasma can be noted only at distances less than $4r_D$ from the disturbing body. Therefore, if the distance between the planes (or the dust grains) exceeds $8r_D$ the potential distribution near each plane can be described by the solution for the semi-infinite plasma. As the experiment shows, the existence of a long-range interaction can be calculated based on a change of the bulk plasma potential, shaped by both planes.

Thus, we can assume that in a dusty plasma the change of the potential on the surface of a single grain causes a change of the bulk plasma potential. But it is valid only for small charges of dust grains, which correspond to weakly coupled plasmas, when the nonideality parameter is less than unity, because the dependence of the bulk plasma potential on the grain surface potential is nonlinear. In strongly coupled plasmas, when the nonideality parameter is greater than unity, the influence of the surface potential fluctuation on the bulk plasma potential decreases. As a result, in a weakly coupled plasma all the dust components can be shaken by a single fluctuation, in the strongly coupled plasma each dust grain oscillates irrespective of the other grains. Thus, in collisional thermal plasmas at atmospheric pressure ordered structures can exist.

[1] J. H. Chu and Lin I, Phys. Rev. Lett. **72**, 4009 (1994).
[2] H. Thomas *et al.*, Phys. Rev. Lett. **73**, 652 (1994).
[3] V. E. Fortov *et al.*, JETP Lett. **64** 92 (1996).
[4] A. A. Samaryan *et al.*, JETP **90**, 817 (2000).
[5] V. I. Vishnyakov and G. S. Dragan, Condens. Matter Phys. **6**, 687 (2003).
[6] I. T. Yakubov and A. G. Khrapak, Sov. Technol. Rev. B **2**, 269 (1989).
[7] V. E. Fortov and I. T. Yakubov, *Nonideal Plasma* (Energoatomizdat, Moscow, 1994).
[8] M. E. Markes and P. F. Williams, Phys. Lett. A **278**, 152 (2000).
[9] S. I. Yakovlenko, Tech. Phys. Lett. **27**, 389 (2001).
[10] E. G. Gibson, Phys. Fluids **2**, 2389 (1966).
[11] V. I. Vishnyakov and G. S. Dragan, Ukr. J. Phys. **49**, 229 (2004).
[12] M. S. Barnes *et al.*, Phys. Rev. Lett. **68**, 313 (1992).
[13] M. Knott and I. J. Ford, Phys. Rev. E **63**, 031403 (2001).
[14] V. I. Vishnyakov and G. S. Dragan, Ukr. J. Phys. **49**, 132 (2004).
[15] *Handbook of Mathematical Functions*, edited by M. Abramowitz and I. A. Stegun, Natl. Bur. Stand. (U.S.) Appl.

Math. Ser. No. 55, (U.S. GPO, Washington, D.C., 1964).
[16] I. S. Grandshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products* (Academic, New York, 1965).
[17] A. E. Dubinov *et al.*, JETP Lett. **25** 73 (1999).
[18] V. A. Gundienkov and S. I. Yakovlenko, JETP **95**, 864 (2002).
[19] V. E. Fortov *et al.*, Phys. Usp. **174**, 495 (2004).
[20] O. Vaulina, S. Khrapak, and G. Morfill, Phys. Rev. E **66**, 016404 (2002).
[21] O. S. Vaulina and S. V. Vladimirov, Phys. Plasmas **9**, 835 (2002).
[22] V. N. Tsytovich, in International Conference on the Physics of Dusty and Combustion Plasmas, Odessa, Ukraine, 2004 [Ukr. J. Phys. (to be published)].