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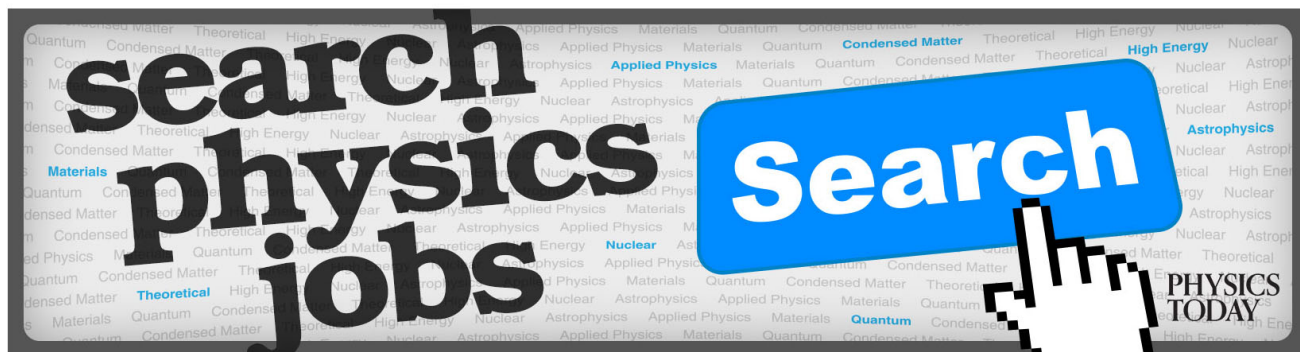
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Attraction of likely charged nano-sized grains in dust-electron plasmas

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Dust-electron plasma, which contains only the dust grains and electrons, emitted by them, is studied. Assumption of almost uniform spatial electrons distribution, which deviates from the uniformity only near the dust grains, leads to the grain charge division into two parts: first part is the individual for each grain “visible” charge and the second part is the common charge of the neutralized background. The visible grain charge can be both negative and positive, while the total grain charge is only positive. The attraction of likely charged grains is possible, because the grain interaction is determined by the visible charges. The equilibrium state between attraction and repulsion of grains is demonstrated. © 2016 AIP Publishing LLC.

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I. INTRODUCTION

Dust-electron plasma consists of positively charged solid or liquid dust grains and electrons, emitted by these grains. Emission can appear as a result of high temperature (thermoemission plasma) or of grain interaction with UV-radiation (photoemission plasma). All the dust grains should be positively charged anyway. However, experimental researches of dust-electron plasma, which is formed in the flame of metallic magnesium block burned in air, demonstrates the existence of negatively charged dust grains.¹ This fact was explained by the assumption that the electron work function of the negatively charged grains exceeds the value as for the positively charged grains. The grain charge is determined by the flux balance: thermionic emission is the electron flux from the dust grain, and a sporadic collision provides the flux of electrons towards the dust grain. As a result, grains with low values of electron work function create the environment saturated with electrons on account of thermionic emission, and acquire positive charges. The high number density of electrons in the environment provides their flux to the grain surface with high electron work function value, which exceeds the thermionic emission flux from this grain. Because of the flux balance requirement, this grain should get a negative charge. It is obvious that the attraction between the dust grains with opposite charges should exist in a system of this kind.

However, the attraction between likely charged grains is also possible in the dusty plasma. For example, it is assumed that in low-pressure gas-discharge complex plasma, which contains dust, electrons, and ions, the plasma particles’ flux towards one dust grain attracts nearby dust grains, creating an additional attraction force between the dust grains—the shadow force.^{2–5} Such a collisionless plasma is described by the orbit motion limited (OML) theory.^{6,7} A force associated with the momentum transfer from the directed ion flux to a dust grain is considered in the OML theory as an ion drag force.^{8–10} The shadow force, which appears because of the

neutral gas component temperature anisotropy, is possible in the nonisothermal systems.²

The attraction between likely charged grains also exists in the strong collisional complex dusty plasma. In this case, interphase interaction provides the ionization balance displacement in the charged dust grain field,¹¹ and this perturbation extends into plasma with a hyperbolic-law decay. As a result, there are anisotropies of the ion and atom number densities around a remote grain, thus creating an extra pressure, which causes the rapprochement of the likely charged dust grains.¹²

In the above considered physical models, the attraction of likely charged dust grains is caused by the forces, which are determined by the ion interaction. In the present paper, the likely charged dust grains’ attraction possibility when the ions are absent, i.e., in the dust-electron plasma, is considered.

II. DUST-ELECTRON PLASMA DESCRIPTION

The description of the dust-electron plasma is a complicated problem. When the electron gas is in equilibrium with the charged dust grains, the Boltzmann distribution law for electrons is valid: $n_e \sim \exp(e\varphi/T)$, where n_e is the electron number density, e is the elementary charge, φ is the electric field potential, and T is the temperature in energy units. In this case, the finite potentials difference corresponds to the electron number densities finite ratio. Therefore, zero electrons number density value at infinity corresponds to zero number density value at the dust grain surface. Otherwise, the condition $n_e(\infty) \rightarrow 0$ is determined by the requirement $\varphi(\infty) \rightarrow -\infty$.

The same problem has been studied by Einstein in cosmology.¹³ To avoid the controversy, he suggested to replace the Poisson equation of the Newton’s gravity law

$$\nabla^2 \phi = 4\pi K \rho$$

by the modified equation

$$\nabla^2 \phi - \lambda \phi = 4\pi K \rho,$$

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where the solution $\phi_0 = -4\pi K\rho_0/\lambda$ corresponds to the uniform distribution of mass in Universe with density ρ_0 . Then, a non-uniformity in the mass distribution should be described by the gravitational potential in the form of $\phi + \phi_0$.

This method was used in Refs. 14 and 15 for dust-electron plasma description. It was suggested that the electron gas is distributed regularly in plasma volume with a uniform number density n_{e0} , and the displacement of electron number density occurs only near the charged dust grains

$$n_e = n_{e0} \exp \frac{e(\varphi - \varphi_0)}{T}, \quad (1)$$

where φ_0 is the bulk plasma potential.

In this case, the potential distribution near the grain surface is described by the Poisson equation in the form

$$\nabla^2 \varphi = 4\pi e(n_e - n_{e0}) = 4\pi e n_{e0} \left(\exp \frac{e\tilde{\varphi}}{T} - 1 \right), \quad (2)$$

where $\tilde{\varphi} = \varphi - \varphi_0$. The condition $n_e \rightarrow 0$ is replaced by the condition $n_e \rightarrow n_{e0}$, under which Eq. (2) is transformed into the Laplace equation, and the tendency $\varphi \rightarrow -\infty$ is replaced by $\varphi \rightarrow \varphi_0$ ($\tilde{\varphi} \rightarrow 0$).

The dust grain charge (here and below the charge is presented in elementary charges), under the condition of neutrality is equal to the number of electrons in the Wigner-Seitz cell,⁶ i.e., in the sphere with radius $R_W = (4\pi n_d/3)^{-1/3}$ around a grain with radius a_j , where n_d is the dust grain number density

$$Z_j = 4\pi \int_{a_j}^{R_W} r^2 n_e(r) dr,$$

whence, with taking Eq. (2) into account, one derives

$$Z_j = (V_W - V_j)n_{e0} + \frac{1}{e} \left(r^2 \frac{d\tilde{\varphi}}{dr} \right) \Big|_{a_j}^{R_W}, \quad (3)$$

where $V_W = 1/n_d$ is the volume of Wigner-Seitz cell, and $V_j = (4/3)\pi a_j^3$ is the dust grain volume. The Wigner-Seitz radius usually $R_W \gg a$, and $\varphi' = 0$ on the cell boundary, therefore, from Eq. (3) it follows:

$$Z_j = \frac{n_{e0}}{n_d} + \frac{1}{e} a_j^2 E_{sj} \equiv Z_0 + \tilde{Z}_j, \quad (4)$$

where $Z_0 = n_{e0}/n_d$ is the background charge, which is created by the uniformly distributed electrons (n_{e0}); E_{sj} is the field near the surface of the j^{th} grain, which in Ref. 14 is

$$E_{sj} = \frac{\sqrt{2}(a_j + r_D)T}{ea_j r_D \text{sgn}(V_{bj})} \sqrt{\exp \frac{V_{bj}}{T} - \frac{V_{bj}}{T} - 1} \quad (5)$$

and, accordingly, the dust grain charge is determined as

$$Z_j = Z_0 + \frac{\sqrt{2}a_j(a_j + r_D)T}{e^2 r_D \text{sgn}(V_{bj})} \sqrt{\exp \frac{V_{bj}}{T} - \frac{V_{bj}}{T} - 1}, \quad (6)$$

where $r_D = (T/4\pi e^2 n_{e0})^{1/2}$ is the screening length.

The potential barrier at the plasma-grain boundary V_b is determined by the electron number density at the grain surface n_{es} to the uniform number density ratio

$$V_b = T \ln \frac{n_{es}}{n_{e0}}. \quad (7)$$

The number density n_{es} is determined by electron flux balance on the grain surface; this balance is provided by the grain charge, which creates the potential barrier at the plasma-grain boundary. One of the fluxes is the electron emission from the dust grain surface, which causes the existence of electron gas in the dust. In thermal plasma, it is the Richardson-Dushman flux

$$I^T = 4\pi a^2 \frac{4\pi m_e(T)^2}{(2\pi\hbar)^3} \exp \frac{-W}{T}, \quad (8)$$

where m_e is the electron mass, \hbar is the Planck constant, and W is the electron work function.

The second flux is determined by the sporadic collisions of electrons with dust grain

$$I^{ads} = \pi a^2 v_{eT} n_{es}, \quad (9)$$

where $v_{eT} = \sqrt{8T/\pi m_e}$ is the electron thermal velocity.

Thus, the surface electron number density in the thermoemission plasma is determined by the flux balance of Eqs. (8) and (9)

$$n_{es} = \nu_e \exp \frac{-W}{T}, \quad (10)$$

and the potential barrier at the plasma-grain boundary

$$V_b = T \ln \frac{\nu_e}{n_{e0}} - W, \quad (11)$$

where $\nu_e = 2(m_e T/2\pi\hbar^2)^{3/2}$ is the effective density of the electron states.

The uniform electron distribution in plasma, with deviation from the value n_{e0} only around the dust grains, leads to the grain charge formal division. The first part, Z_0 , is constant for all dust grains and is determined only by the electron uniform number density and grain number density. The system neutrality condition is $Zn_d = \bar{n}_e$, where Z is the average dust grain charge and \bar{n}_e is the average electron number density. Then, equality $Z_0 n_d = n_{e0}$ should be considered as a condition of background neutralizing, i.e., the uniformly distributed electrons are neutralized by the part of grains' charge Z_0 .

Electron number density deviation near the grain surface is described by the second part of the grain charge \tilde{Z}_j , whose average value is specified by the equality $\tilde{Z} n_d = \bar{n}_e - n_{e0}$. The field at the grain surface is determined only by this part of the charge, as follows from Eq. (4): $E_{sj} = e\tilde{Z}_j/a_j^2$.

It can be assumed that there exists a certain distribution of the grain charges. Then, the situation that the total charge of a k^{th} dust grain is less than background charge is possible: $Z_0 > Z_k > 0$, i.e., the own part of charge $\tilde{Z}_k < 0$ and the field created by this grain $E_{sk} < 0$. In the case, when at the same

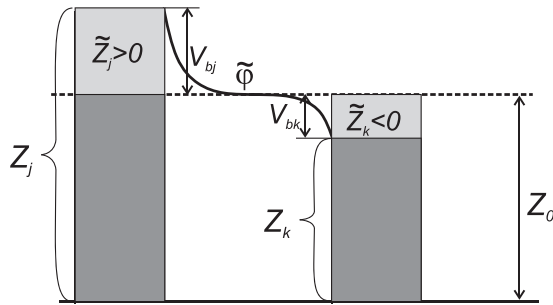


FIG. 1. Possible potential distribution between the likely charged dust grains.

time the neighboring dust grain has the total charge $Z_j > Z_0$, the potential is distributed between these dust grains according to a monotonous function (Fig. 1), and there is an attractive force between these grains. Note that the total charges of both dust grains are positive, but these are attracted.

It should be noted that interacting dust grains “do not know” about the existence of the background neutralized charge, as their interaction is determined only by the part of charge \tilde{Z} , which can be called a “visible” part of the charge. If an observer was inside this system, he could detect only the visible part of the charge, but not the neutralized charge. It is similar to a situation with dark matter in modern cosmology. In Ref. 16, it is noted: “We can conclude that although we may possess measurements ∇u and $\nabla^2 u$, we cannot determine the nature of the scalar field u simply from the Poisson equation or Gauss’ Theorem.”

III. CHARGE DISTRIBUTION OF THE DUST GRAINS

The dust in the thermal plasma forms as a result of vapor condensation. Such origin of the dust grains means that their size distribution is lognormal^{17,18}

$$f(d) = \frac{n_d}{d \ln \sigma \sqrt{2\pi}} \exp \frac{-(\ln d - \ln d_0)^2}{2 \ln^2 \sigma}, \quad (12)$$

where d is the grain diameter, $d_0 = 2\bar{a} \exp(-\ln^2 \sigma/2)$ is the median of the distribution, and σ is the standard deviation.

The grain size distribution causes distribution of grains by charges. First, the grain charge depends on its size via Eq. (6); second, for nano-sized grains, there exists a relationship between the electron work function and the grain radius¹⁹

$$W(a) = W_0 + \frac{0.39e^2}{a}. \quad (13)$$

The average grain charge is determined by the neutrality condition $Zn_d = \bar{n}_e$, where the average electron number density is determined by the Boltzmann distribution

$$\bar{n}_e = n_{e0} \exp \frac{e\varphi_0}{T} \quad (14)$$

and the bulk plasma potential is determined by the Coulomb energy per electron²⁰ and the neutralized charge Coulomb energy per dust grain¹⁵

$$\varphi_0 = \frac{3e}{2n_d} \left(Z_0^2 n_d^{4/3} - \bar{n}_e^{4/3} \right) \quad (15)$$

in respect that $\sqrt[3]{4\pi/3} \cong 3/2$.

Considering $n_{e0} = Z_0 n_d$, the equation for the neutralized charge can be obtained from Eqs. (14) and (15)

$$Z(Z_0) = Z_0 \exp \left[\frac{3e^2}{2T} n_d^{1/3} \left(Z_0^2 - Z(Z_0)^{4/3} \right) \right], \quad (16)$$

where $Z(Z_0)$ is the average value of the total charge Eq. (6).

When Eq. (16) is solved with respect to Z_0 , it is possible to obtain other parameters, which describe the plasma. Let the dust grains with average radius $a = 1$ nm and the number density $n_d = 10^{12} \text{cm}^{-3}$ be in equilibrium with a neutral buffer gas at the atmospheric pressure and under Kelvin temperature $T/k_B = 3000$ K (k_B is the Boltzmann constant). Suppose that electron work function is equal to $W_0 = 4.3$ eV, as is for some widespread metals (for example, Fe, Al, and Mo). Under these conditions, the thermionic emission from the grains provides the average electron number density of $\bar{n}_e = 3.1 \times 10^{12} \text{cm}^{-3}$, uniform number density of $n_{e0} = 3.0 \times 10^{12} \text{cm}^{-3}$, and neutralized charge of $Z_0 = 3$. The average grain radius increase up to $a = 5$ nm leads to the neutralized charge growth up to $Z_0 = 6.8$, the average electron number density up to $\bar{n}_e = 8.6 \times 10^{12} \text{cm}^{-3}$, and the uniform number density up to $n_{e0} = 6.8 \times 10^{12} \text{cm}^{-3}$. Suppose that dust grain size distribution is described by Eq. (12) with standard deviation $\sigma = 2$. These distributions are presented in Fig. 2 for two average grain radii $a_1 = 1$ nm and $a_2 = 5$ nm.

Potential barrier of Eq. (11) is a function of the dust grain radius, in accordance with Eq. (13). Then, dependence of the visible charge on the grain radius $\tilde{Z}(a)$ is determined not only by the linear function Eq. (6), but also by Eq. (13). The resulting relationships of \tilde{Z} and grain diameter are presented in Fig. 2. The function $\tilde{Z}(d)$ has a minimum in the

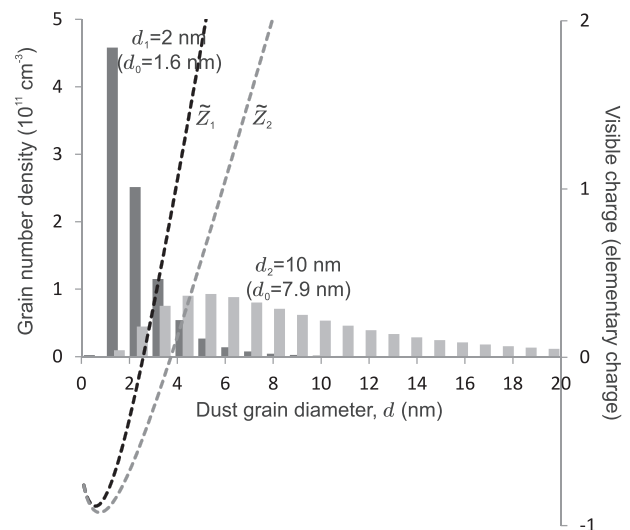


FIG. 2. Dust grain size distributions and dependencies of visible charges \tilde{Z} on the grain diameter in the thermal dust-electron plasma for the average grain diameters $d_1 = 2$ nm (black) and $d_2 = 10$ nm (grey).

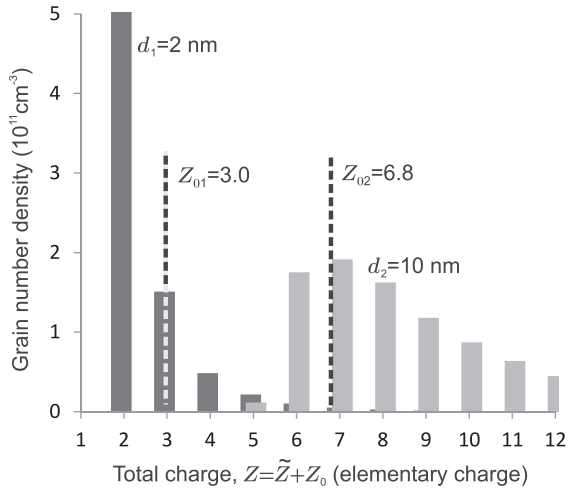


FIG. 3. Charge distributions of dust grains in the thermal dust-electron plasma for the average grain diameters $d_1 = 2 \text{ nm}$ (black) and $d_2 = 10 \text{ nm}$ (grey).

range of small sizes, i.e., the dust grains with different sizes can have equal charges, which must be taken into account in the grain charge distribution. These charge distributions are presented in Fig. 3. The existence of the minimum in the dependence $\tilde{Z}(d)$ causes the increase of grains' part in the range of small charges.

Thus, the dust grains with both positive and negative visible charges can exist in the thermal dust-electron plasma simultaneously. As it follows from distributions shown in Fig. 3, the great part of the dust grains has the total charge $Z < Z_0$. The situation presented in Fig. 1 is implemented for interaction of these grains and grains with total charge $Z > Z_0$.

The neutralized charge is $Z_0 = 3$ for the dust grains with diameter 2 nm, i.e., the grains with total charge $Z = 3$ have no potential barrier V_b . The grains with total charge $Z = 2$ have a negative potential barrier, and the grains with total charge $Z \geq 4$ have a positive potential barrier. The increase in the uniform number density n_{e0} with increasing the average grain diameter up to 10 nm leads to a potential barrier decrease (see Eq. (11)) and, accordingly, of the visible charge \tilde{Z} . In this case, the neutralized charge is $Z_0 \sim 7$, i.e., the grains with total charge $Z = 7$ have no potential barrier; the grains with $Z = 5$ and $Z = 6$ have a negative potential barrier; and the grains with $Z \geq 8$ have a positive potential barrier. The monotonous spatial distribution of the potential between the dust grains means that the field does not change direction and these grains are attracted.

IV. DISCUSSION

The different charges of dust grains, even with the same chemical composition, are caused by the grain size distribution and by dependence of electron work function on the grain size. Such a dependence is essential only for nano-sized dust grains; accordingly, only dust grains of this kind should be attracted by the mechanism, which is considered above. This mechanism can be observed at the vapor

condensation, when nuclei with sub-nanometer sizes are formed in the environment with dust, which was condensed earlier.

For example, metal oxide vapor condensation occurs at the homogeneous burning of metalized fuel without the alkali addition agents. Such a system can be considered as a dust-electron plasma. As it was demonstrated in Ref. 21, the large number of liquid nuclei causes their Brownian coalescence and the formation of large liquid grains. In the meantime, the thermodynamics of the system requires the presence of equilibrium nuclei number; and the bimodal size distribution of liquid grains (droplets) occurs at the initial stage of nucleation: the first mode average diameter is $\sim 2 \text{ nm}$, and the second mode average diameter is $\sim 10 \text{ nm}$. These grains have identical chemical composition, but their electron work functions, defined by Eq. (13), are different. In this case, emergence of the above-described situation is possible, and the grains of different modes will attract each other.

Formation of nano-sized dust is possible in the cosmic vacuum as a result of gas condensation. These dust grains can be charged under the solar UV-radiation by the photoelectric emission. The charged grains and the electrons emitted by them create the dust-electron plasma. In this case, the electron-emission flux equals²²

$$I^{ph} = \pi a^2 Y_{j_{ph}} \exp \frac{-V_b}{T}, \quad (17)$$

where Y is the quantum yield, j_{ph} is the photon flux density with the photon energy higher than the electron work function

$$j_{ph}(a) = \frac{2\pi T_S^3}{c^2 h^3} \int_{w(a)/T_S}^{\infty} \frac{x^2 dx}{\exp(x) - 1}. \quad (18)$$

$T_S/k_B = 5778 \text{ K}$ is the solar Kelvin temperature.

The potential barrier at the plasma-grain boundary is determined by the balance of fluxes (Eqs. (9) and (17))

$$V_b = T \ln \frac{Y_{j_{ph}}}{n_{es} v_{eT}} = \frac{T}{2} \ln \frac{Y_{j_{ph}}}{n_{e0} v_{eT}}, \quad (19)$$

where the Kelvin dust grain temperature is $T/k_B \sim 280 \text{ K}$.²³

In this case, there also exists the dependence of the grain charge on the electron work function, which limits the photon flux of Eq. (18). Thus, the mechanism considered above is applicable for the cosmic dust, and attraction of the likely charged grains is possible.

The existence of the attraction force between the dust grains provides their rapprochement. However, the mechanism considered above is applicable only in the case if there are free electrons between the interacting grains, which create the neutralized background. In the opposite case, the neutralized background is absent and grains' repulsion due to the Coulomb interaction of their total charges occurs. For the dust grain approach description, the uniform number density should be determined as a function of the distance between grains

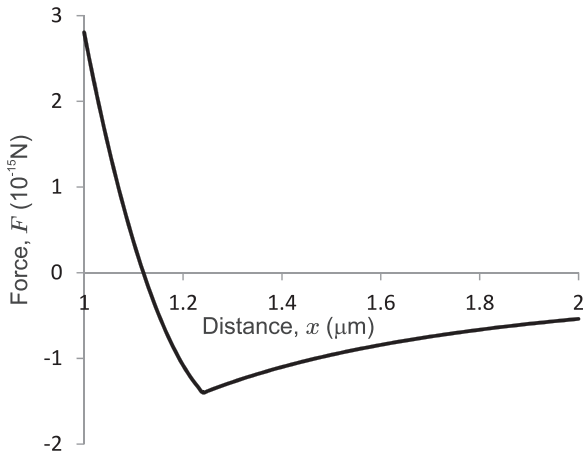


FIG. 4. Dependency of interaction force on the distance between grains.

$$n_0(x) = \begin{cases} n_{e0}V_x n_d, & V_x < 1/n_d \\ n_{e0}, & V_x \geq 1/n_d, \end{cases} \quad (20)$$

where $V_x = \pi x^3/6$ is the volume per one grain determined by the distance between the grains x .

Then, the neutralized charge between interacting grains is determined as $z_0(x) = n_0(x)/n_d$; and the visible grain charge is the function of the distance between them: $\tilde{z}(x) = Z - z_0(x)$, where Z is the total charge, that does not change under interaction. Thus, the interaction force can be determined in the form

$$F(x) = \frac{e^2[Z_1 - z_0(x)][Z_2 - z_0(x)]}{x^2}, \quad (21)$$

i.e., repulsion of dust grains occurs at short distances between them, because $z_0(x) \rightarrow 0$ in this case, and attraction of the grains occurs at long distances, because in this case $z_0(x) = Z_0$.

Suppose two dust grains with radii $a_1 = 1$ nm and $a_2 = 7$ nm allocated in the thermal dust-electron plasma considered above, with average grain radius 5 nm. In this case, $Z_0 = 6.8$, and the total grain charges are $Z_1 = 5$ and $Z_2 = 12$. The dependence of the force Eq. (21) on the distance between grains is presented in Fig. 4. The equilibrium is reached at the distance x_{eq} , when the potential barrier Eq. (11) of the first grain $V_{b1} = 0$, i.e.,

$$x_{eq} = \left[\frac{6\nu_e}{\pi n_{e0} n_d} \exp \frac{-W(a_1)}{T} \right]^{1/3} \cong 1.1 \mu\text{m}.$$

The initial dust grain number density in the considered example is $n_d = 10^{12} \text{ cm}^{-3}$. The grains' approach to the equilibrium distance provides the local increase in the number density up to $1.4 \times 10^{12} \text{ cm}^{-3}$. Thus, the self-organization and formation of dust grain ordered structures²⁴ is possible in the dust-electron plasma.

V. CONCLUSION

The dust grains' interaction in the dust-electron plasma is determined only by the visible part of the total grain charge, which is defined as a difference between the total

grain charge and the neutralized background charge. Therefore, the dust grains with likely total charges can be attracted if their visible charges have opposite signs. The existence of total charges' different values of grains with identical chemical composition is caused by the grain size distribution and by the dependence of electron work function on the grain size. This dependence is essential only for nano-sized dust grains; accordingly, only such dust grains should be attracted by the considered mechanism.

Such an effect can be observed in the vapor condensation, when nuclei with sub-nanometer sizes are formed in the environment with dust, which was condensed earlier. It can be a condensation of hot vapor in the atmosphere, or the gas condensation in cosmic vacuum under solar UV-radiation. The electron work function of nuclei is much higher than of the grains formed earlier, that stimulates the large charge difference and, accordingly, causes the grains' attraction.

The attraction force between the dust grains leads to their approach. However, the considered mechanism is applicable only until free electrons are present between the interacting grains, which create the neutralized background. In the opposite case, the neutralized background is absent, and the grains' repulsion due to the Coulomb's interaction of their total charges occurs. Therefore, there exists an equilibrium distance between the two interacting dust grains, in which the local neutralized background provides zero visible charge of one of the grains. The existence of equilibrium in the dust grains' interaction means that self-organization of dust-electron plasma is possible.

¹V. I. Vishnyakov, G. S. Dragan, and A. V. Florco, *J. Exp. Theor. Phys.* **106**, 182 (2008).

²V. N. Tsytovich, *Phys.-Usp.* **40**, 53 (1997).

³U. de Angelis, A. Forlani, and G. Masiello, *Phys. Plasmas* **7**, 3198 (2000).

⁴K. Avinash, *Phys. Plasmas* **8**, 2601 (2001).

⁵D. Samsonov, A. V. Ivlev, G. E. Morfill, and J. Goree, *Phys. Rev. E* **63**, 025401(R) (2001).

⁶V. E. Fortov, A. G. Khrapak, S. A. Khrapak, V. I. Molotkov, and O. F. Petrov, *Phys.-Usp.* **47**, 447 (2004).

⁷J. Goree, *Plasma Source Sci. Technol.* **3**, 400 (1994).

⁸A. M. Ignatov and S. G. Amiranashvili, *Phys. Rev. E* **63**, 017402 (2000).

⁹G. Morfill and V. N. Tsytovich, *Phys. Plasmas* **9**, 4 (2002).

¹⁰S. Khrapak, S. Ratinskaia, A. Zobnin, M. H. Thoma, M. Kretschmer, A. Usachev, V. Yaroshenko, R. A. Quinn, G. E. Morfill, O. Petrov, and V. Fortov, *Ukr. J. Phys.* **50**, 151 (2005).

¹¹V. I. Vishnyakov and G. S. Dragan, *Phys. Rev. E* **73**, 026403 (2006).

¹²V. I. Vishnyakov, *Phys. Plasmas* **12**, 103502 (2005).

¹³A. Einstein, "Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie," in *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften* (Sitzungsberichte, Berlin, 1917), p. 142.

¹⁴V. I. Vishnyakov and G. S. Dragan, *Phys. Rev. E* **74**, 036404 (2006).

¹⁵V. I. Vishnyakov, *Phys. Rev. E* **85**, 026402 (2012).

¹⁶J. P. Baugher, *Prog. Phys.* **1**, 15 (2013).

¹⁷C. G. Granqvist and R. A. Buhrman, *J. Appl. Phys.* **47**, 2200 (1976).

¹⁸A. A. Ennan, S. A. Kiro, M. V. Oprya, and V. I. Vishnyakov, *J. Aerosol Sci.* **64**, 103 (2013).

¹⁹B. M. Smirnov, *Sov. Phys. Usp.* **35**, 1052 (1992).

²⁰S. Ichimaru, *Rev. Mod. Phys.* **54**, 1017 (1982).

²¹V. I. Vishnyakov, S. A. Kiro, and A. A. Ennan, *J. Aerosol Sci.* **67**, 13 (2014).

²²V. E. Fortov, A. P. Nefedov, O. S. Vaulina, A. M. Lipaev, V. I. Molotkov, A. A. Samaryan, V. P. Nikitskii, A. I. Ivanov, A. V. Kalmykov, A. Ya. Solov'ev, and P. V. Vinogradov, *J. Exp. Theor. Phys.* **87**, 1087 (1998).

²³I. Mann, in *Proceedings of the 7th Spacecraft Charging Technology Conference* (European Space Agency, Noordwijk, The Netherlands, 2001), p. 629.

²⁴V. N. Tsytovich, *Phys.-Usp.* **58**, 150 (2015).