Electron and ion number densities in the space charge layer in thermal plasmas

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The space charge layer near the charged surface in the thermal collision plasma has been studied. The analytical expressions describing the spatial distributions of electron and ion number densities, taking into account the nonequilibrium ionization, have been obtained. It has been shown that the nonequilibrium ionization in the space charge layer leads to the essential change of the distribution profile of these number densities. © *2006 American Institute of Physics*. [DOI: 10.1063/1.2186528]

I. INTRODUCTION

There is a potential barrier in the boundary layer when the plasma comes in contact with an electrode or a dust grain surface, and the number densities of the charge carriers inside this layer can greatly change if compared to their values in the volume of the plasma. The assessment of these number densities is necessary for the understanding of many phenomena of interface interaction. In particular, it allows defining the floating potential of the electrode or the charge of the dust grain[1–4] and the parameters of the grains oscillations.[5,6] The knowledge of the number densities' spatial distribution is necessary for the definition of forces of the interface interaction, causing the formation of the ordered structures in the plasma.[7,8]

Many investigations dedicated to the definition of the spatial distribution of charge carrier number densities in the space charge area,[9–15] with major attention paid to the numerical modeling, have been carried out. The present paper is dedicated to the analytical determination of the electron and ion number densities near the flat electrode in the collision thermal plasma.

II. STATE OF PROBLEM

The low-temperature thermal plasma represents a gas at the atmospheric pressure and at the temperatures around $T=0.1-0.3$ eV$(T/k_B=1200-3500$ K). It usually contains easily ionizable atoms of alkali metals as natural impurity or in the form of the special additional agents with number density $n_A \sim 10^{16}-10^{23}$ m^{-3}, which are the basic suppliers of free electrons and positive singly charged ions. The ionization of the additional agent's atoms in the thermal plasma occurs due to the collisions between the gas particles; therefore, such plasma is strongly collisional, unlike the low-pressure gas-discharge plasma. The temperatures of the plasma components are close to each other and the system can be considered as isothermal.

The equilibrium ionization in the thermal plasma, which does not contain the dust component, is described by the Saha equation

$$\frac{n_e n_i}{n_a} = \frac{g_i}{g_a} \nu_e \exp\left(\frac{-I}{T}\right) \equiv K_S, \tag{1}$$

where $\nu_e = 2(m_e T/2\pi\hbar^2)^{3/2}$ is the effective density of the electron states, g_i and g_a are the statistical weights of ions and atoms, I is the ionization potential of the alkali-metal atoms, T is the equilibrium temperature of the plasma, which in this case is an isothermal system, m_e is the electronic mass, $\hbar$ is the Planck constant, and K_S is the Saha constant.

It is true, provided the charge and mass are preserved, that

$$n_e = n_i = n_0, \quad n_i + n_a = n_A, \tag{2}$$

where n_0 is the unperturbed number density, which can change from 10^{10} to 10^{21} m^{-3}; and in the low-temperature plasma the ionization degree is low, i.e., $n_i \ll n_a \sim n_A$.

The interaction between the plasma and the electrode or the charged dust grains leads to the emergence of the volume charge of plasma whose average value is characterized by the bulk plasma potential φ_{pl}, and for the flat electrode it is defined by the following expression:[16]

$$\varphi_{pl} = -2\frac{T}{e}\tanh\left(\frac{e\phi_s}{4T}\right), \tag{3}$$

where $e\phi_s$ is the potential barrier at the interface boundary.

There is a change of ionization degree of plasma and the Saha equation needs to be modified,[17]

$$\frac{n_q^2}{n_A} = K_S \exp\left(\frac{-e\varphi_{pl}}{T}\right), \tag{4}$$

where $n_q = \sqrt{n_e n_i}$ is the quasiunperturbed number density.

Everywhere in plasma, except for the space charge layer near the charged surface, the equilibrium is maintained, and the electron and ion number densities can be described by the Boltzmann distribution law with respect to the quasiunperturbed number density

$$n_e = n_q \exp\left(\frac{e\phi}{T}\right), \quad n_i = n_q \exp\left(\frac{-e\phi}{T}\right), \tag{5}$$

where the potential ϕ is counted off from the bulk plasma potential.

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Inside the space charge layer the equilibrium is disturbed and Eq. (5) does not hold as the nonequilibrium charge carriers appear. The formation of the nonequilibrium charge carriers is considered in detail in Ref. 18. In the present paper the results of this investigation for the buildup of the spatial distributions of the electron and ion number densities inside the space charge layer near the surface of the electrode are used.

III. ELECTRON AND ION SPATIAL DISTRIBUTIONS AT SMALL POTENTIALS

In each point of the space charge layer there is some value of electrical potential and field, which provides, in accordance with Ref. 18, for the presence of the nonequilibrium charge carriers in this point,

$$\delta n = n_q \frac{\exp(e\phi/T) - 1}{1 + (er_D E/T)^2}\left(1 + \frac{\lambda_R}{r_D}\right), \tag{6}$$

where λ_R is the length of nonequilibrium charge carriers recombination and $r_D = \sqrt{\varepsilon_0 T/2e^2 n_q}$ is the screening length. In the thermal plasma at atmospheric pressure the inequality $\lambda_R \ll r_D$ is fulfilled; therefore the term λ_R/r_D can be neglected. It is necessary to note that Eq. (6) is suitable only for the plasma at atmospheric and higher pressure. For example, at a pressure of 0.1 atmospheres $\lambda_R \sim r_D$ and solutions of the equations in Ref. 18 must be others.

Thus, the full electron and ion number densities are described by the following expressions:

$$n_e = n_{e0} + \delta n, \quad n_i = n_{i0} + \delta n, \tag{7}$$

where n_{e0} and n_{i0} are the equilibrium values of the number densities, defined by the Boltzmann distribution law [Eq. (5)]; and as we shall see further, for the nonequilibrium additive the following relation is true: $\delta n \ll n_q$.

From Eqs. (5)–(7), taking into account that $1 + (er_D E/T)^2 = 2\cosh(e\phi/T) - 1$, we obtain

$$n_e(r) = n_q \frac{\exp[2e\phi(r)/T]}{2\cosh[e\phi(r)/T] - 1}, \tag{8}$$

$$n_i(r) = n_q \frac{\exp[-2e\phi(r)/T] + 2\sinh[e\phi(r)/T]}{2\cosh[e\phi(r)/T] - 1}. \tag{9}$$

The spatial distribution of potential is obtained as the solution of the Poisson-Boltzmann equation. The presence of nonequilibrium charge carriers can change the distributions of the potential and the field. This change is taken into account by the field of ambipolar diffusion of the nonequilibrium charge carriers.[18]

In case of the small potentials, when $|e\phi| \ll T$, Eqs. (8) and (9) are reduced to the following form:

$$n_e(r) \cong n_q[1 + 2e\phi(r)/T], \quad n_i \cong n_q. \tag{10}$$

The distributions of the electron and ion number densities in the space charge layer at small value of the potential are presented in Fig. 1. The distributions have been constructed for the flat surface, located in the plasma, containing the additional agent of potassium ($I = 4.3$ eV) with the number density $n_A = 10^{20}$ m^{-3} at the temperature $T = 1.5$ eV. In these conditions the equilibrium parameters are as follows: $n_0 \sim 6 \times 10^{17}$ m^{-3}, $\lambda_R \sim 0.1\mu$, and $r_D \sim 3\mu$.

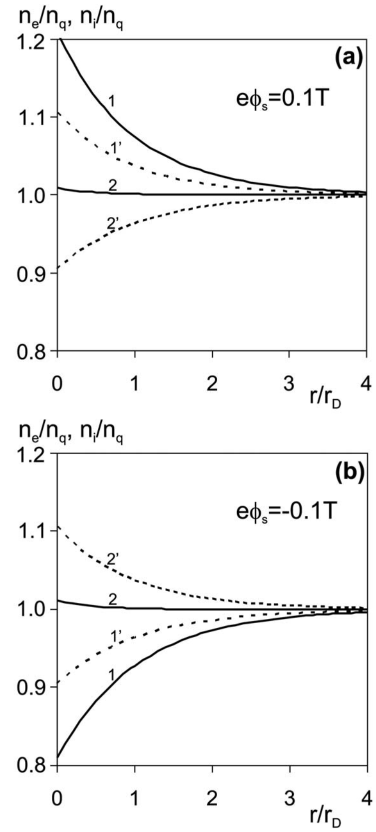

FIG. 1. The distributions of the (1) electron and (2) ion number densities at small value of the surface potential: (a) is the positive potential; (b) is the negative potential. Curves (1′) and (2′) are the Boltzmann distributions.

The presence of the nonequilibrium ionization leads to insignificant spatial change of the ion number density and it remains equal approximately to n_q. For comparison, the results[10] of numerically solving the boundary problem of grain charging in a nuclear-induced Xe plasma with grain number density 10^{12} m^{-3} at temperature $T/k_B = 300$ K are represented in Fig. 2. Profiles 1 and 3 are calculated by the establishment method, the symbols show some of the numerous points obtained by the relaxation method, and profiles 2 and 4 are computed using the approximate theory of grain

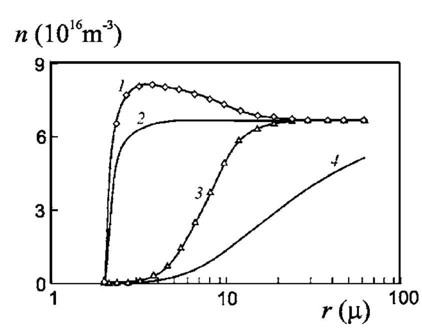

FIG. 2. The distributions of (1, 2) ions and (3, 4) electrons near the dust grain surface at the x-ray's ionization action (Ref. 10).

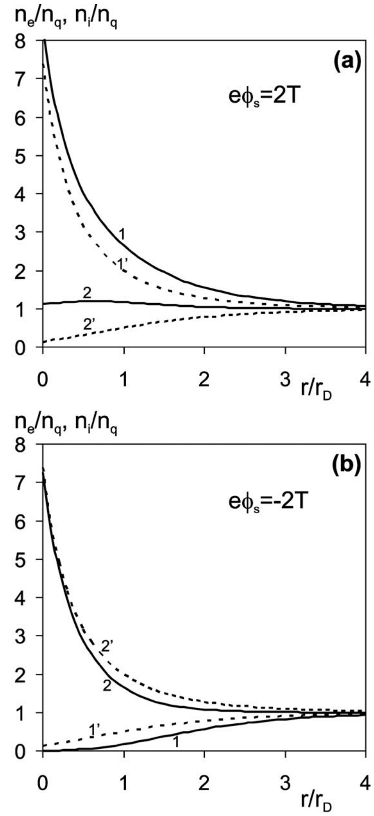

FIG. 3. The distributions of the (1) electron and (2) ion number densities at large value of the surface potential: (a) is the positive potential; (b) is the negative potential. Curves (1′) and (2′) are the Boltzmann distributions.

charging. As it is seen, these results well coincide with the results, obtained by taking into account the nonequilibrium ionization in the space charge layer.

Such behavior of the ion number density should affect the formation of the interface potential barrier and the forces responsible for the formation of the ordered structures in plasma.

IV. ELECTRON AND ION SPATIAL DISTRIBUTIONS AT LARGE POTENTIALS

The identical change of the electron and ion spatial distributions at the positive and negative surface potentials is the property of small values of the potential only. When the inequality $|e\phi| \ll T$ does not hold, the distributions become different for different signs of the potential. In Fig. 3(a) the spatial distributions of electron and ion number densities for the positive potential, when $e\phi > T$, are presented. The presence of nonequilibrium charge carriers leads to the increase of the electron and ion number densities; the increase of the ion density due to the nonequilibrium carriers surpasses the decrease of their density due to the electrical forces of repulsion. The tendency, observable at small potentials, is preserved.

In Fig. 3(b) the spatial distributions of electron and ion number densities for the negative potential, when $-e\phi > T$ are shown. In this case there is a decrease of the electron and ion densities in relation to the Boltzmann distribution law. However, the decrease of the ion density at the potential $\phi_s = -2T/e$ is not so significant as to compensate for the electrical attraction. It is explained by the fact that the resulting electron number density cannot be less than zero, which limits the nonequilibrium charge carriers' density at the negative surface potential. The change of the density δn is assumed identical for the electrons and ions because for the one-charging ions each act of the recombination is accompanied by the disappearing of one ion and one electron.

For the extreme cases of larger magnitude of the positive and negative potentials it is possible to simplify Eqs. (8) and (9) as follows:

(i) For the positive potential,

$$n_e(r) \cong n_q \exp\left[\frac{e\phi(r)}{T}\right],$$
$$n_i(r) \cong n_q, \quad e\phi_s \gg 0. \tag{11}$$

(ii) For the negative potential,

$$n_e(r) \cong n_q \exp\left[3\frac{e\phi(r)}{T}\right],$$
$$n_i(r) \cong n_q \exp\left[-\frac{e\phi(r)}{T}\right], \quad e\phi_s \ll 0. \tag{12}$$

At $e\phi \gg 0$ the electron number density tends to the Boltzmann distribution law. It is connected with the fact that the change of electron density as a result of nonequilibrium ionization is much less than the equilibrium value $\delta n \sim n_q \ll n_{e0}$. The ion density, as well as at small values of the potential, change a little in the space charge layer and is close to the quasiunperturbed number density.

It should be noted that at large magnitude of the potential, there is no necessity for calculation of the bulk plasma potential as, according to Eq. (3), at $|e\phi_s| \gg T$ the hyperbolic tangent tends to one; therefore, $e\varphi_{pl} \cong -2T\,\mathrm{sgn}(\phi_s)$. Accordingly, for the positive potential $n_{q+} = n_0 \exp(1)$, and for the negative potential $n_{q-} = n_0 \exp(-1)$.

At $e\phi \ll 0$ there is an essential difference from the case of small potentials, namely, the ion density is no more equal n_q, but tends to the Boltzmann distribution law. It is connected with the fact that the decrease of the ionization degree in the low-temperature plasma, when $n_i \ll n_a$, cannot be arbitrarily high; that is why as the electron density near the surface of the negative electrode decreases, and δn decreases too.[18] Thus the electron number density decreases three times as fast, if compared to its equilibrium value.

V. CONCLUSION

Thus, it is possible to draw a conclusion that the nonequilibrium ionization changes the form of the spatial distributions of electrons and ions. Thus, it is not only the external action in the form of radiation,[10] but also the presence of the charged surface that are the sources of the nonequilibrium ionization. The surface of dust grains, for example, is such a surface. Therefore, it is arguable that the presence of the dust grains in the plasma leads to the essential change of the ionization state of plasma and the occurrence of spatial inhomogeneity of the ionization degree.

Besides, when electric current passes through plasma, one electrode has the positive potential, the other – negative. Consequently, the nonequilibrium ionization appears: being more equilibrium near the positive electrode, and less near the negative electrode. Hence, there is the diffusion of the nonequilibrium charge carriers along the streamlines, i.e., the electric current, passing through the plasma, is accompanied by the diffusion of nonequilibrium charge carriers.

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